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A cost-benefit analysis of moose harvesting in Scandinavia.
A stage structured modelling approach

Abstract

A cost-benefit analysis of moose (*Alces alces*) harvesting in Scandinavia is presented within the framework of an age structured model with four categories of animals (calves, yearlings, adult females and adult males). The paper aims to demonstrate the economic content of such a wildlife model and how this content may change under shifting economic and ecological conditions. Two different harvesting regimes are explored: landowner profit maximization, where the combined benefit of harvesting value and browsing damage is taken into account, and overall management, where the cost and damages of moose-vehicle collisions are taken into account as well. An empirical analysis of the Norwegian moose stock indicates that the present stock level is far too high compared with the overall management scenario, and that the composition of the harvest could be improved.

Key words: moose hunting, cost-benefit analysis, stage model
JEL: Q28

1. Introduction

The aim of this paper is twofold: first, to demonstrate the economic content of an age structured wildlife population model; and second, to show how this economic content may change under different management scenarios. The wildlife considered is the moose (*Alces alces*) which is studied in a Scandinavian ecological and institutional context where the landowners obtain the harvesting value and bear the cost of the timber browsing damage, but do not pay for possible other damages. Two basic management schemes are analysed; landowner management and overall management where the cost of moose-vehicle collisions is taken into account as well.

Analysing structured wildlife harvesting models, i.e., models where the species are grouped in different classes according to age and sex, has a long tradition within biology. Caswell (2001) gives an in-depth overview; see also Getz and Haigh (1989). However, economic analysis plays a minor role in these works. Economic reasoning is taken into account in Skonhoft et al. (2002) who analysed various management strategies for a mountain ungulate living in a protected area and a hunting area. Four stages were included: females and males within and outside the protected area. However, because of the complexity of this model due to the dispersal mechanism it is difficult to understand the various economic mechanisms influencing harvesting and abundance.

The present paper aims to analyse such economic mechanisms more explicitly where a four-stage model (calves, yearlings, adult females and adult males) is formulated. Ericsson et al. (2000) studied the Swedish moose harvest policy with respect to selective versus random harvest of the different stages. In their simulations, however, they only accounted for hunting profit. Wam and Hofstad (2007) also studied a stage structured moose model in a Scandinavian context. The land owner profit was maximized and the trade-off between meat value and timber browsing damage was considered. Such trade-off will also be analysed here, but, as indicated, traffic damage costs will be taken into account as well. These costs are quite high, and recent estimates indicate that they may be even higher than that of the moose meat value (see below). Another important difference compared to the Wam and Hofstad study is that our model, at least to some extent, is solved analytically. We are thus able to show more directly the driving forces behind the harvesting composition and the various harvesting scenarios. We find that per animal values (meat value plus omitted damage value due to harvesting) are instrumental in determining the optimal harvesting composition. The similarity with the results in the seminal Reed (1980) paper is apparent. In addition, we explicitly model a female-calf harvest restriction as the current code of conduct among hunters prevent that calves are left without their mother their first winter (section four). A novelty of our paper is thus to demonstrate the analytical and numerical consequences of imposing such restriction. As in Wam and Hofstad (2007) the model is illustrated numerically where the Norwegian moose stock is used as an example. Just as in Ericsson et al. (2000), we also calculate the benefit of our selective harvesting scheme with a harvest pattern where ‘an animal is an animal’ as considered in the traditional bioeconomic analysis (e.g., Clark 1990).

The paper is organized as follows. In the next section, moose hunting in Scandinavia is briefly described. In section three the population model is formulated while section four demonstrates what happens when the hunting is steered by the traditional landowner goal of maximizing meat value. The landowner exploitation is analyzed both with and without including the browsing damage cost. In section five we study the optimal sex and age composition as well as the economic consequences when the harvest is steered by the overall manager, and where the traffic damage cost, in addition to the meat value and browsing damage cost, are taken into account. Section six illustrates the models by numerical simulations using Norwegian aggregate data and where the various scenarios are compared with recent harvest and stock data. In the basic model, the meat value is assumed to be given by a fixed meat price, and the unit costs related to forest damage and traffic accidents are assumed to be constant as well. In section 6.3 these assumptions are relaxed and we show some numerical results when stock dependent hunting costs as well as convex forest damage costs are included. Section seven finally summarizes our findings.

2. Moose hunting in Scandinavia

The moose is the world’s largest member of the deer family and is found in the northern forests of North America, Europe and Russia. It is by far the most important game species in Scandinavia, and in Norway and Sweden about 35,000 and 100,000 animals, respectively, are shot every year. The value of this harvest is substantial, and the meat counts for more than 2 per cent of the yearly meat consumption in these countries. The moose hunting, which takes place in September and October, is also an important cultural event in a large number of local communities. Moose hunting has traditionally been a local activity, and landowners receive the hunting value. The hunters have been the local people; the landowners and their families and friends, and the traditional management goal has been to maximize the meat value, possibly corrected for forest browsing damage, for stable populations, i.e., biological equilibrium (more details are provided in Skonhoft and Olaussen (2005) and the references therein).

The moose stock and moose harvest in Scandinavia have increased substantially since the 1950-60's when about 4-7000 moose were shot annually in Norway while it counted about 10-30000 in Sweden (see Figure 1). The explanation for these increases is twofold. First, selective harvesting of different stages of the moose population has increased the biomass production considerably. The main principle is to harvest more bulls, yearlings and calves and less of the productive female stage to increase the meat biomass produced. Second, changing the forestry practice from selective logging to clear cutting has increased the forage production and availability (Saether et al. 1992). Note especially the extreme increase in Sweden during the late seventies and early eighties when harvest of calves first was introduced (Ericsson et al. 2000). The rapid increase in the moose population was strengthened by the fact that (especially in Sweden) the hunters came short of harvesting the reproduction in the late seventies. This changed during the eighties and the decrease in the stock reflects that the hunters kept harvesting more than the reproduction for some years.

Figure 1 about here

Recently, more attention has been paid to the increased costs associated with the high density of moose. The estimates of the annual forest damage cost due to lost forest production in Sweden vary from 400 to 1050 million NOK annually (Skogforsk 2009).[1] Based on the relative moose densities in Norway and Sweden, these estimates indicate that the yearly forest damage cost in Norway may be in the range of NOK

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[1] 500 to 1300 million SEK = 410 to 1066 million NOK (exchange rate 0.82 May 09).

160-400 million. However, the Norwegian forest is less productive and one calculation indicates significant lower damage cost (Solbraa 1998) while a more recent estimate indicates the yearly damage to be about 70 million NOK (Larsen 2007, see also Storaas et al. 2001). In addition, traffic accidents due to collisions with cars and trains are costly. On average, during the period 2002-2007, 1350 and 750 moose have been killed annually by car and train, respectively, in Norway. Based on Solstad (2007), the yearly total cost of moose collisions in Norway is about 200 million NOK. The number of car-moose incidents in Sweden is between 4500-5000 annually, and the associated cost is estimated to be about 500 million NOK (Ingemarson et al. 2007).

As mentioned, the traditional exploitations scheme has been steered by the aim of maximising the meat production. This is hereafter denoted the landowner regime LO because the landowners receive the hunting value of the meat. Depending on the migration pattern, the landowners may or may not be concerned with the browsing damage which varies in different regions (see Skonhoft and Olaussen 2005 for details). We therefore distinguish between two stylized situations; the first regime LO where landowners neglect any potential browsing damage, and the other regime LOF where landowners take forest damage into account, and analyse these two as separate cases. In addition, we compare these regimes with an overall management scheme OM, where both forest damages and traffic accident costs, in addition to the meat value benefit, are taken into account.

3. Population model

The *Alces alces* is a large ungulate with mean slaughter body weight (about 55% of live weight) for adult moose in Scandinavia of about 170 kg for males and 150 kg for females. The non-harvest mortality rates are generally low due to lack of predators, and there is no evidence of density-dependent mortality. On the other hand, fecundity has proven to be affected by the female density while the number of males seems to be of negligible importance within the range of moose densities in Scandinavia. However, it may play a role if the sex composition is very skew (see, e.g., Nilsen et al. 2005 and the references therein for more details).

Just as in Lande et al. (2003) the population at time (year) t is structured in four stages; calves $X_{c,t}$ ($yr < 1$), yearlings $X_{y,t}$ ($1 \leq yr < 2$), adult females $X_{f,t}$ ($yr \geq 2$) and adult males $X_{m,t}$ ($yr \geq 2$) so that the total moose population is $X_t = X_{c,t} + X_{y,t} + X_{f,t} + X_{m,t}$. The population is measured in spring before calving. All stages are generally harvested, and the hunting, as already indicated, occurs in September–October (see Figure 2). Natural mortality is assumed to take place during the winter, after the hunting season, as the natural mortality throughout summer and fall is small and negligible. The same natural mortality rate is imposed for males and females, and, as mentioned, the same sex ratio is assumed for the yearlings when they enter the adult stages.

Figure 2 about here

Neglecting any stochastic variations in biology and environment, and any dispersal in and out of the considered area, the number of calves (recruitment) is first governed by $X_{c,t} = r_t X_{f,t}$ with $r_t = r(X_{f,t}, X_{m,t})$ as the fertility rate (number of calves per female). The fertility rate generally depends on both female density (number of females) and male density. It decreases in the female density $\partial r / \partial X_{f,t} = r'_f < 0$, and may also be reduced when the number of males become low, $r'_m \geq 0$. In addition, we have $r(0, X_{m,t}) > 0$ when $X_{m,t} > 0$ and $r(X_{f,t}, 0) = 0$. Therefore, the recruitment function yields:

(1) $$X_{c,t} = r(X_{f,t}, X_{m,t}) X_{f,t}.$$

The recruitment function, starting from the origin, may be a one-peaked value function in the female density, but intuitively $\partial X_{c,t} / \partial X_{f,t} = (r + r'_f X_{f,t}) > 0$ should hold in an optimal harvesting programme, at least when females represent meat hunting value and no positive stock value is included. If not, less female hunting coincides with less recruitment which barely can represent an efficient harvest strategy. This is confirmed by the numerical simulations (section six).

The number of yearlings follows next as:

(2) $$X_{y,t+1} = s_c (1 - h_{c,t}) X_{c,t}$$

where s_c is the fixed calf natural survival rate (fraction) and $0 \le h_{c,t} < 1$ is the calf harvesting rate. Finally, the abundance of (adult) females and (adult) males become:

(3) $$X_{f,t+1} = 0.5 s_y (1 - h_{y,t}) X_{y,t} + s(1 - h_{f,t}) X_{f,t}$$

and

(4) $$X_{m,t+1} = 0.5 s_y (1 - h_{y,t}) X_{y,t} + s(1 - h_{m,t}) X_{m,t} ,$$

respectively, and where the same sex ratio is assumed for the yearlings when they enter the adult stages. s is the fixed natural survival rate, identical for females and males while s_y is the yearling survival rate. $0 \le h_{y,t} < 1$, $h_{f,t}$ and $h_{m,t}$ are the harvesting rates of yearlings, females and males, respectively. While the calf and yearling harvesting rates must be less than one to omit stock depletion, the harvest rates of the adult females and males can in principle equalize one. However, if $h_{m,t} = 1$ and $h_{f,t} = 1$ we find that the adults in the long term will be individuals of age two only. This may lead to evolutionary drift (Solberg et al. 2009 and references therein). For this reason, we restrict the adult harvest rates to be below one, i.e., $0 \le h_{m,t} < 1$ and $0 \le h_{f,t} < 1$.

The population model (1) – (4) has a recursive structure, and when combining (1) and (2) we find:

(5) $$X_{y,t+1} = s_c (1 - h_{c,t}) r(X_{f,t}, X_{m,t}) X_{f,t} .$$

Therefore, equations (3) – (5) is a reduced form model in three stages and where all equations are first order difference equations. This form is used when studying the different exploitation schemes below. The system may, however, be reduced further when omitting the yearling population. When combining equations (5) and (3), the female population growth hence reads $X_{f,t+1} = 0.5 s_y (1 - h_{y,t}) s_c (1 - h_{c,t-1}) r(X_{f,t-1}, X_{m,t-1}) X_{f,t-1} + s(1 - h_{f,t}) X_{f,t}$ while combining equations (5) and (4) yields the male population growth as $X_{m,t+1} = 0.5 s_y (1 - h_{y,t}) s_c (1 - h_{c,t-1}) r(X_{f,t-1}, X_{m,t-1}) X_{f,t-1} + s(1 - h_{m,t}) X_{m,t}$. These two equations represent a system of two interconnected second order non-linear difference equations, and numerical analyses demonstrate that the equilibrium is unique and stable for fixed harvesting rates (see e.g., Gandolfo 2001 for a theoretical exposition).

Omitting the time subscript, the female equilibrium (when $X_f > 0$ and $X_m > 0$) reads:

(6) $$1 = 0.5 s_y (1 - h_y) s_c (1 - h_c) r(X_f, X_m) + s(1 - h_f)$$

while

(7) $$X_m = 0.5 s_y (1 - h_y) s_c (1 - h_c) r(X_f, X_m) X_f + s(1 - h_m) X_m$$

represents the male population equilibrium. The female equilibrium condition (6) slopes upward in the $X_m - X_f$ plane, but becomes flat when of X_m is 'large' as the number of males then has a negligible fertility effect. Higher harvesting rates h_c, h_y and h_f shift it down. The male equilibrium equation (7) slopes upward as well, but will bend backwards for 'large' values of X_f when $(r + r'_f X_f)$ becomes negative (see also above).[2] As indicated, it will be a unique biological equilibrium for fixed harvesting rates (see Figure 3). Higher harvesting rates h_c, h_y and h_m shift the male equilibrium condition inwards. Therefore, not surprisingly, more aggressive harvesting of all stages reduces the female as well as the male density. On the other hand, the yearlings abundance $X_y = s_c (1 - h_c) r(X_f, X_m) X_f$ increases under higher harvesting pressure if the harvesting activity initially is modest and the intersection of the male and female equilibrium conditions takes place on the backward bending part of the male equilibrium schedule. The same will be so for the equilibrium calf population, $X_c = r(X_f, X_m) X_f$. On the contrary, if the intersection between the male and female equilibrium schedules takes place on the upward sloping part of the male schedule, increased harvesting activity reduces all stages of the population.

Figure 3 about here

When combining the above equilibrium equations (6) and (7), the male–female proportion may be written as $X_m / X_f = [1 - s(1 - h_f)]/[1 - s(1 - h_m)]$. Therefore, the yearling harvest (as well as the calf harvest) does not influence the equilibrium adult population proportion because the natural mortality of males and females is equal, and the same fraction of yearlings enters the female and male populations. It is also seen

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[2] The slope of the female population equilibrium (6) is given by $dX_f / dX_m = -r'_m / r'_f$ while $dX_f / dX_m = [1 - s(1 - h_m) - 0.5 s_y (1 - h_y) s_c (1 - h_c) X_f r'_m] / [0.5 s_y (1 - h_y) s_c (1 - h_c)(r + r'_f X_f)]$ is for the male equilibrium (7).

that sex-skewed adult harvesting rates translate directly into sex-skewed adult abundance. In an unexploited system, we have $X_m / X_f = 1$ together with equation (6) as $r(X_f, X_m) = (1-s)/0.5 s_y s_c$.

4. Landowner exploitation

4.1 The cost and benefit functions

We first study the situation where the landowner, or the group of landowners acting as a single agent through a landowner association, maximises the meat value while taking the timber browsing damage cost into account. This is the LOF regime (section two above). Because natural mortality basically occurs during the winter, after the hunting season (see above), the number of animals removed in the different categories in year t are $H_{c,t} = h_{c,t} X_{c,t}$, $H_{y,t} = h_{y,t} X_{y,t}$, $H_{f,t} = h_{f,t} X_{f,t}$ and $H_{m,t} = h_{m,t} X_{m,t}$. In this regime, as well as in the LO regime where the browsing damage costs are neglected and the overall management regime OM, the current hunting value is accordingly:

$$(8) \qquad Q_t = p(w_c h_{c,t} X_{c,t} + w_y h_{y,t} X_{y,t} + w_f h_{f,t} X_{f,t} + w_m h_{m,t} X_{m,t}).$$

$w_c < w_y < w_f < w_m$ are the (average) body slaughter weights (kilogram per animal) of the four stages and p (NOK per kilogram) the hunting price, assumed to be similar for all stages. The unit hunting price is supposed to be independent of the amount harvested and the stock sizes. When further assuming that the marginal cost of the landowners of organizing this hunting is stock independent, the harvest price p is a 'net' price assumed to be fixed.[3] As mentioned, this assumption stock independent costs is relaxed in section 6.3.

As already indicated, the forest browsing damage on pine trees occur during the winter when other food sources are restricted. We assume this damage to be related to the number of animals, determined by the population sizes during the winter and approximated by the population sizes after the hunting, $D_i((1-h_{i,t})X_{i,t})$

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[3] Following Scandinavian practices, hunters pay a licence from the landowner. One licence allows the hunter to shoot one animal (e.g., one yearling) but is paid only if the animal is killed. The final payment is a payment per kilo of the actual moose shot. Wam and Hofstad (2007) also applied a fixed per kg meat price across stages. Hussain and Tschirhart (2010) describe a far more complicated licence-price system for elk hunting in the US while Naevdal et al. (2010) look at a situation of Scandinavian moose hunting where adult males are shot for their trophy value and where the price is contingent upon the number of animals hunted and 'quality', as expressed by the abundance of males in the population.

($i = c, y, f, m$).[4] It is further assumed that more animals mean more damage, $D_i(0) = 0$ and $D_i^{'} > 0$. Following Wam and Hofstad (2007) the damage functional forms are generally unknown and will vary due to circumstances, like the quality of the timber stands, the timber market conditions, the productivity of the forest and so forth, and can be concave as well a convex functions (or even concave – convex, or convex – concave).[5] For this reason, we simply choose a compromise and approximate these costs by linear functions as in Ingemarson et al. (2007) (but see section 6.3).

With linear functions, the current browsing damage hence reads:

(9) $$D_t = d_c(1-h_{c,t})X_{c,t} + d_y(1-h_{y,t})X_{y,t} + d_f(1-h_{f,t})X_{f,t} + d_m(1-h_{m,t})X_{m,t}$$

and where $d_c < d_y < d_f = d_m$ are the per animal (NOK per animal) cost which differ among the stages, just as in Wam and Hofstad (2007). As no more cost and benefit components are included for the landowner, the current landowner profit is $(Q_t - D_t)$ in the LOF regime and simply Q_t in the LO regime. Henceforth, only the LOF maximization problem is analyzed as the LO is completely analogous with $D_t = 0$.

4.2 The optimality conditions

The present-value landowner profit is to be maximised subject to the above biological constraints. In addition to these constraints, a female - calf harvest restriction is included because the common hunting code of conduct is to never shoot an adult female without also harvesting her calf.[6] This restriction is included simply as:

(10) $$h_{f,t}X_{f,t} \le h_{c,t}X_{c,t},$$

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[4] The average stock sizes over the winter possibly describes the actual browsing damage better, but comes at the cost of considerable notational clutter without altering the qualitative aspect of the model. How to decide on this and, and similar questions, is an inherent problem of time discrete models.

[5] In general, the browsing may also have additional components as it can influence native ground flora and biodiversity.

[6] As mentioned, calving takes place early spring (May) while harvest takes place in the autumn (September - October). (See Figure 2). Since the calves are dependent on their mother the first winter, this female – calf harvest restriction is also implemented in several local harvest regulation plans (Solberg 2009, see also e.g. Glomfjord-Spildra Storvald 2009). Surprisingly, this important feature of the management practice is neglected in both Nilsen et. al. (2005) and Lande et al. (2001).

indicating that the harvest of the adult females never can exceed the calf harvesting. This restriction must always hold in any harvest scheme. The LOF problem is hence to $\max\limits_{h_{c,t},h_{y,t},h_{f,t},h_{m,t}} \sum_{t=0}^{t=\infty} \rho^t (Q_t - D_t)$ subject to the growth equations (3) – (5), and (10), and where $\rho = 1/(1+\delta)$ is the discount factor with $\delta \geq 0$ as the (yearly) discount rate. The Lagrangian of this problem may be written as

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} \rho^t \Big\{ & \Big[p\Big(w_c h_{c,t} r(X_{f,t}, X_{m,t}) X_{f,t} + w_y h_{y,t} X_{y,t} + w_f h_{f,t} X_{f,t} + w_m h_{m,t} X_{m,t} \Big) \Big] \\
& - [d_c (1-h_{c,t}) r(X_{f,t}, X_{m,t}) X_{f,t} + d_y (1-h_{y,t}) X_{y,t} + d_f (1-h_{f,t}) X_{f,t} + d_m (1-h_{m,t}) X_{m,t}] \\
& - \rho \eta_{t+1} \Big[X_{y,t+1} - s_c (1-h_{c,t}) r(X_{f,t}, X_{m,t}) X_{f,t} \Big] \\
& - \rho \lambda_{t+1} \Big[X_{f,t+1} - 0.5 s_y (1-h_{y,t}) X_{y,t} - s(1-h_{f,t}) X_{f,t} \Big] \\
& - \rho \mu_{t+1} \Big[X_{m,t+1} - 0.5 s_y (1-h_{y,t}) X_{y,t} - s(1-h_{m,t}) X_{m,t} \Big] - \rho \omega_{t+1} \Big[h_{f,t} X_{f,t} - h_{c,t} r(X_{f,t}, X_{m,t}) X_{f,t} \Big] \Big\}
\end{aligned}
$$

and where $\eta_t > 0$, $\lambda_t > 0$, $\mu_t > 0$ and $\omega_t \geq 0$ are the shadow prices of the yearling, female and male populations, and the female – calf harvesting restriction, respectively. Notice that the population equation shadow prices, in contrast to the female – calf constraint shadow price, always will be strictly positive.

All the first-order control conditions of this maximizing problem are stated with the actual complementary slackness conditions and where the possibility for keeping each of the stages unexploited is considered while harvesting whole sub populations are not possible (section three above). These control conditions then read:[7]

(11) $$\frac{\partial L}{\partial h_{c,t}} = rX_{f,t}(pw_c + d_c - \rho\eta_{t+1}s_c + \rho\omega_{t+1}) \leq 0 \ ; \ 0 \leq h_{c,t} < 1,$$

(12) $$\frac{\partial L}{\partial h_{y,t}} = X_{y,t}(pw_y + d_y - \rho\lambda_{t+1}0.5s_y - \rho\mu_{t+1}0.5s_y) \leq 0 \ ; \ 0 \leq h_{y,t} < 1,$$

(13) $$\frac{\partial L}{\partial h_{f,t}} = X_{f,t}(pw_f + d_f - \rho\lambda_{t+1}s - \rho\omega_{t+1}) \leq 0; \ 0 \leq h_{f,t} < 1$$

and

(14) $$\frac{\partial L}{\partial h_{m,t}} = X_{m,t}(pw_m + d_m - \rho\mu_{t+1}s) \leq 0 \ ; 0 \leq h_{m,t} < 1.$$

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[7] As the recruitment function may not be concave in the female abundance when the number of females becomes 'large', we have a potential non-convexity problem in our optimization. However, as already indicated (section three), such large female population can not represent an optimal harvesting policy. See also numerical section below.

The state, or portfolio conditions, with $X_{y,t} > 0$, $X_{f,t} > 0$ and $X_{m,t} > 0$, are:

(15)

$$\frac{\partial L}{\partial X_{y,t}} = pw_y h_{y,t} - d_y(1-h_{y,t}) - \eta_t + \rho\lambda_{t+1}0.5s_y(1-h_{y,t}) + \rho\mu_{t+1}0.5s_y(1-h_{y,t}) = 0\,,$$

(16)

$$\frac{\partial L}{\partial X_{f,t}} = pw_c h_{c,t}(r + r'_f\, X_{f,t}) + pw_f h_{f,t} - d_f(1-h_{f,t}) + \rho\eta_{t+1}s_c(1-h_{c,t})(r + r'_f\, X_{f,t}) - \lambda_t$$
$$+\rho\lambda_{t+1}s(1-h_{f,t}) - \rho\omega_{t+1}[h_{f,t} - h_{c,t}(r + r'_f\, X_{f,t})] = 0$$

and

(17) $$\frac{\partial L}{\partial X_{m,t}} = pw_c h_{c,t} r'_m\, X_{f,t} + pw_m h_{m,t} - d_m(1-h_{m,t}) + \rho\eta_{t+1}s_c(1-h_{c,t})r'_m\, X_{f,t}$$
$$-\mu_t + \rho\mu_{t+1}s(1-h_{m,t}) + \rho\omega_{t+1}h_{c,t}r'_m\, X_{f,t} = 0$$

Condition (11) says that calf harvest should take up to the point where marginal harvest income plus the benefit of reduced browsing damage is equal to, or below, the cost in term of reduced yearling growth evaluated at its shadow price while also taking the discount rent into account. The female - calf harvesting restriction shadow price is also included. When this condition holds as an inequality, the marginal benefit is below its marginal cost and harvesting of this stage is thus not profitable, $h_{c,t} = 0$. In a similar manner, condition (12) indicates that harvesting of the yearlings should take place up to the point where the marginal benefit is equal to, or below, the cost in terms of reduced population of males and females. The female condition (13) is simpler as reduced stock growth works only through its own stage. The shadow price of the female - calf harvesting constraint is also included here, indicating this as an additional shadow cost. The male condition (14) is analogous to the female harvesting condition. As discussed (section three), it is assumed that $h_{m,t}$ is less than one as well.

The yearling stock portfolio condition (15) may also be written as $\eta_t = pw_y h_{y,t} - d_y(1-h_{y,t}) + \rho\lambda_{t+1}0.5s_y(1-h_{y,t}) + \rho\mu_{t+1}0.5s_y(1-h_{y,t})$. It hence indicates that the number of yearlings should be maintained so that its shadow price equalizes the meat value minus the browsing damage of one more animal on the margin plus its growth contribution to the adult stages evaluated at their shadow prices and when discounting is taken into account as well. The other stock portfolio conditions (16) and (17) are more complex, but can be given similar interpretations. When rewriting (17), we hence find that the value of the male shadow price is $\mu_t = pw_c h_{c,t} r'_m X_{f,t} + pw_m h_{m,t} - d_m(1-h_{m,t}) + \rho\eta_{t+1} s_c(1-h_{c,t}) r'_m X_{f,t}$ $+\rho\mu_{t+1} s(1-h_{m,t}) + \rho\omega_{t+1} h_{c,t} r'_m X_{f,t}$. It is therefore seen that a binding female – calf constraint and $\omega_{t+1} > 0$ partially works in the direction of a higher male stock shadow price .

These first order conditions together with the biological constraints comprise a complex dynamic system with eleven unknowns and eleven equations. It is therefore difficult, if not impossible, to assess the dynamics analytically. However, because of the linear cost and benefit functions, some important tradeoffs that occur when composing the harvest can be studied. Furthermore, we may also ask if it is likely that all stages should be harvested, and which stages that are most likely to be exploited in the optimal solution.

Before trying to answer these questions, it should first be noted that harvests of the calf and yearling stages may be seen as substitutes because both activities reduce the female and male stock. Intuitively, we may suspect that yearling harvest is more beneficial than calf harvest because yearlings have a higher harvesting value (weight) and generate more damage (see also below). However, on the other hand, since it is a constraint linking calf harvest to female harvest, we may find that giving up an inefficient calf harvest means that we also give up the potentially beneficial female harvest. Hence, if the cost of giving up female harvest is high, the optimum programme would possibly involve some calf harvest as well. Whether this occurs or not is clearly an empirical question depending on parameter values. We start to analyze these questions in the simplified situation when the female – calf constraint (10) is assumed not to bind. This unbinding case sheds light over the case when the restriction binds. This unbinding case may also be important in the management of other species where such code of conduct restriction is irrelevant, i.e., for many small game species and in fisheries.

4.3 Unbinding female- calf harvest constraint, $\omega_t = 0$

With the female - calf constraint not binding, $\omega_t = 0$, we first ask whether it is more likely to harvest only yearlings, females or males, two of these stages, or all stages. We find that yearling harvest will contradict the optimality conditions. To see this, suppose first that yearling, but also female and male harvest, take place; that is, $h_{y,t} > 0$, $h_{f,t} > 0$ and $h_{m,t} > 0$. All the control conditions (12) – (14) then holds as equations. Combining these equations (with positive stock sizes) yields $pw_y + d_y = 0.5(s_y / s)(pw_f + d_f + pw_m + d_m)$. As only parameters are included, this equation will hold only by accident. Therefore, harvesting all these three stages at the same time can not represent an optimal solution when the female - calf harvesting constraint is not binding.

Suppose next that we have positive yearling harvest together with zero female and positive male harvest, i.e., $h_{y,t} > 0$, $h_{f,t} = 0$ and $h_{m,t} > 0$. Condition (13) then holds as an inequality and combination of conditions (12) – (14) yields therefore $pw_y + d_y > 0.5(s_y / s)(pw_f + d_f + pw_m + d_m)$. Note first that if we assume $w_y = w_f = w_m$ and $d_y = d_f = d_m$ this inequality holds suggested that $s_y < s$. Furthermore, if $w_y < w_f < w_m$ and $d_y < d_f \leq d_m$, and additionally if s_y is sufficiently lower than s, the inequality still holds. However, in the specific case of moose hunting in Scandinavia considered here, this outcome seems very unlikely. Therefore, since the adult and yearling survival rates s and s_y (‘biological discount rates’) are more or less identical (Table 1, numerical section), and the yearling weight is below that of the females, which again is below that of males, $w_y < w_f < w_m$, and the marginal browsing damage of adults typically is above that of the yearlings, $d_y < d_f = d_m$ (Table 1), we find that the above inequality represents a contradiction. Note that the hunting price (NOK per animal) and the size of the browsing damage work in the same direction in this reasoning. Note also that there are the cost/price ratios that matter for the outcome, not the price nor the absolute (marginal) costs.

In a similar manner, we find that positive yearling harvest together with zero male and positive female harvest yields the same inequality as above and hence also contradict the optimality conditions for the given structure of parameter values. The assumption of positive yearling harvest together with zero male and zero female harvest contradicts the optimality conditions as well. We can therefore conclude that positive yearling harvest is not a possible option when the female - calf restriction is not binding under the assumption of constant harvest price and constant marginal browsing damage for all categories of animals. On the other hand, we find that zero yearling harvest together with positive harvest of one, or both, of the adult categories do not contradict the optimality conditions. The reason for this outcome is simply that the per animal values (meat value plus omitted damage value due to harvesting) are higher for the adults than the yearling category while the survival rates do not differ too much. The similarity with the findings in the seminal Reed (1980) paper is apparent.[8]

Above we argued that yearling harvest is more likely to be beneficial than calf harvest since they are close substitutes in the harvest while the yearling harvest value at the same time is higher. Now, under the the unbinding adult female - calf constraint (10), it is also possible to show that positive calf harvest can not be optimal suggested that there is no yearling harvest. With zero yearling harvest $h_{y,t} = 0$, the yearling state condition (15) writes $\eta_t + d_y = \rho\lambda_{t+1}0.5s_y + \rho\mu_{t+1}0.5s_y$, while the yearling control condition (12) is $pw_y + d_y < \rho\lambda_{t+1}0.5s_y + \rho\mu_{t+1}0.5s_y$. These two conditions yield $\eta_t > pw_y$. When combining the calf control condition (11) under the assumption of positive calf harvest and $\omega_t = 0$ with the above inequality, we find $(1/\rho s_c)(pw_c + d_c) > pw_y$, or $(pw_c + d_c)/s_c > \rho pw_y$, i.e., a higher 'biological discounted' meat plus omitted damage value of the calf age class than the economic discounted meat value of the yearlings. When $w_y = w_c$, this inequality may hold. Furthermore, if $(pw_c + d_c) < pw_y$ and additionally s_y and $\rho = 1/(1+\delta)$ are sufficiently low, this inequality may still hold. This outcome is, however, not likely because of the substantial more valuable harvest of the yearling than the calf category together with the small calf damage cost and the high calf survival rate (again, see Table 1 numerical section). Therefore, suggested that there is no harvest of the yearling stage and if the discount factor is not too low, i.e. the discount rate δ is not too high, it can

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[8] Reed finds that that the differences in the weight – survival ratio (or the 'biological discounted biomass content') of the various harvestable age classes determines the optimal fishing composition.

not be beneficial to harvest the calf population when the female - calf harvest constraint is not binding[9].

4.4 Binding female – calf harvest restriction, $\omega_t > 0$

The above analysis shows that if the female – calf restriction does not bind, there will be adult harvesting only. From this analysis we may also suspect that whether both or only one of the adult stages should be harvested will depend on parameter values, like the animal weights and the survival rates. However, if both females and males are harvested in the unbinding case, this result may change when the restriction binds. The reasoning goes as follows. If females are harvested, the code of conduct restriction (10) indicates that at least the same number of calves must be harvested. Since the restriction is unbinding if more calves than females are harvested, we already know from the unbinding case that no calves should be harvested. Therefore, harvesting more calves than females represents a contradiction both in the unbinding as well as the binding case. Hence, the only remaining case to be considered is the case when the restriction binds; that is, the number of calves and females harvested are similar, or zero.

When constraint (10) binds, it will certainly also influence the various possibilities of yearling harvest together with adult female and male harvest. We may hence find that yearling harvest together with male harvest occurs in the optimal solution. The intuition is straightforward since the harvest of females implies an extra cost in terms of reducing all other stages as more calves are harvested than in the unrestricted case. This extra cost may turn the optimal harvest scheme in favour of only male and yearling harvest. Hence, when the female harvest is restricted by the calf harvest constraint, yearling harvest may not longer be less valuable than adult harvest. Because yearlings and calves can be seen as substitutes in terms of their effects on reducing male and female stocks, we may also find that male harvest together with calf and hence adult female harvest, without yearling harvest, can represent the optimal strategy.

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[9] For the parameter values in Table 1, $(pw_c + d_c)/s_c > \rho p w_y$ does not hold if $\rho > 0.56$, or $\delta < 0.78$ (78%).

It can also be shown analytically that zero yearling harvest together with calf harvest is less likely to represents a contradiction when the female-calf constraint binds. To see this, we again combine the yearling state condition (15) and the yearling control condition (12) and the calf control condition (11), now under the assumption of a binding female – calf harvest constraint. The result is $(pw_c + d_c + \rho\omega_{t+1})/s_c > \rho pw_y$ which now may hold due to the positive shadow price of the female – calf constraint. In the numerical analysis, we also find calf harvest without yearling harvest to be included in the optimal solution. Sensitivity analyses also indicate that this holds in the range of realistic parameter values. Again, we stress that all conclusions about the harvesting composition is based on the linear damage cost and benefit assumptions (but see section 6.3 and Appendix).

4.5 The LO regime

In the LO regime where the browsing damage is not taken into account, we find the same optimality conditions as the above conditions (11) - (17), except that the marginal damage term is missing. Therefore, if the female – calf harvesting constraint is not binding, and based on realistic parameter values, we reach the same harvest composition conclusions in this regime as in the LOF regime because the damage cost structure is similar the marginal harvest benefit structure; that is, $d_c < d_y < d_f \le d_m$ and $w_c < w_y < w_f < w_m$. This means that we will still have adult harvesting only, just as explained in the LOF case above (section 4.3).

However, with the female - calf constraint binding, we may reach different conclusions compared to the LOF regime because the value of the female – calf constraint shadow price changes when the browsing damage cost components are not included. Indeed, this is actually what happens under the baseline parameter value scenario (see numerical section) where only yearlings and adult males are harvested. The intuition is exactly as above because the additional cost of harvesting females is that it requires the same amount of calf harvest. It turns out that when neglecting forest damage costs, yearling and male harvest is preferred to harvest combinations involving female and calf harvest. The crucial parameters here are the weights of the different stages as the average weight of a female and calf is sufficiently lower than that of a yearling (see Table 1, numerical section). Hence, harvesting yearlings generates more benefits than harvesting adult females together with calves.

5. Overall optimality

So far the harvest decision has been determined by the landowner, or the landowner association acting as a single agent, with and without taking browsing damage into account. However, to assess the overall value of the moose harvest, more cost and benefit components should be included. As already mentioned, the single most important of these is the damage related to moose-vehicle and moose-railway collisions. These costs can be considerable (section two) and are not experienced by the landowners. A simple, yet realistic way to account for this type of damage cost is, just as for the browsing damage, to relate it to the population density, and where again, *ceteris paribus*, a higher density means more damage. As most of the traffic incidents happen during the winter, this cost component is also approximated by the population size after harvesting (but before natural mortality). Again, the cost functions depend on circumstances, like weather and snow conditions, and may be concave as well a convex in the number of animals, but are still approximated by using linear functions (see also Ingemarson et al. 2007):

(18) $$T_t = t_c(1-h_{c,t})X_{c,t} + t_y(1-h_{y,t})X_{y,t} + t_f(1-h_{f,t})X_{f,t} + t_m(1-h_{m,t})X_{m,t} \ .$$

t_i ($i = c, y, f, m$) is therefore the per animal (NOK per animal) cost which, just as for the browsing damage cost, varies among the different stages. The current overall surplus hence reads $(Q_t - D_t - T_t)$.[10]

The overall management problem (OM regime) is then to

$$\max_{h_{c,t}, h_{y,t}, h_{f,t}, h_{m,t}} \sum_{t=0}^{t=\infty} \rho^t (Q_t - D_t - T_t)$$, again subject to the above biological growth equations (3) – (5), together with the female – calf harvest constraint (10). It is easily recognized that the necessary first order conditions of this problem will be just as above except

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[10] Note that the term overall optimality considered here is meant in a restrictive manner and should not be confused with a social optimal solution in the traditional meaning. We ignore both non-consumptive and some consumptive benefits and costs of the moose stock such as existence value, wildlife viewing, fear of wildlife, crop damage costs, biodiversity effects, and so forth.

that the additional traffic damage terms are included. The control conditions hence read:

(19) $$\frac{\partial L}{\partial h_{c,t}} = rX_{f,t}(pw_c + d_c + t_c - \rho\eta_{t+1}s_c + \rho\omega_{t+1}) \le 0 \ ; \ 0 \le h_{c,t} < 1,$$

(20) $$\frac{\partial L}{\partial h_{y,t}} = X_{y,t}(pw_y + d_y + t_y - \rho\lambda_{t+1}0.5s_y - \rho\mu_{t+1}0.5s_y) \le 0 \ ; \ 0 \le h_{y,t} < 1,$$

(21) $$\frac{\partial L}{\partial h_{f,t}} = X_{f,t}(pw_f + d_f + t_f - \rho\lambda_{t+1}s - \rho\omega_{t+1}) \le 0 \ ; \ 0 \le h_{f,t} < 1$$

and

(22) $$\frac{\partial L}{\partial h_{m,t}} = X_{m,t}(pw_m + d_m + t_m - \rho\mu_{t+1}s) \le 0 \ ; 0 \le h_{m,t} < 1.$$

As harvest reduces the traffic damage cost, just as it reduces the browsing damage cost, and because the marginal damage generally increases with the size of the animals; that is, we have $t_c < t_y < t_f \le t_m$ (again, see numerical section), the structure of the solution of this problem will be as above, both when the female - calf restriction (10) binds and when it does not. Hence, when it binds, the same number of calf and adult females, either positive or zero, will be hunted in the optimal solution. We may suspect that the harvest of the different stages and the harvest composition changes due to a shift in the female- calf harvest constraint shadow price. However, as the shift works in the same direction as the shift imposed by forest damages, it seems reasonable that the harvest composition will follow more or less the same pattern as in the LOF case. Moreover, just as in biomass models (e.g., Skonhoft and Olaussen 2005), we may also suspect that the harvest will be higher and the number of animals lower when more damage costs are included.

6. Numerical illustration

6.1 Data and specific functional forms

The exploitation will now be illustrated numerically. The fertility rate, decreasing in the number of females, is specified as a sigmoidal function with an increasing degree of density dependence at high densities (Nilsen et al. 2005). The function reads:

(23) $$r_t = r(X_{f,t}, X_{m,t}) = \frac{\tilde{r}}{1 + (X_{f,t}/K)^b}(1 - e^{-aX_{m,t}})$$

with $\tilde{r} > 0$ as the intrinsic growth rate (maximum number of calves per female). The fertility rate shifts down with a smaller male – female population ratio and where $a > 0$ yields this composition ratio effect. $K > 0$ is the female stock level for which density-dependent fertility is equal to density-independent fertility. Thus, for a stock level above *K*, density-dependent factors dominate. This parameter scales the population sizes, and its value is contingent upon factors like the size of the area, the biological productivity of the grazing and browsing resources, and so forth. The compensation parameter $b > 0$ indicates to what extent density-independent effects compensate for changes in the stock size.

Equation (23) implies a recruitment function $X_{c,t} = r(X_{f,t}, X_{m,t})X_{f,t} = \tilde{r}(1-e^{-aX_{m,t}})X_{f,t}/[1+(X_{f,t}/K)^b]$ and is of the so-called Shepherd type. With $b > 1$, which is assumed, we have the Ricker version meaning that $\partial X_{c,t}/\partial X_{f,t} < 0$ for a high female density. However, as already indicated (section three), $\partial X_{c,t}/\partial X_{f,t} > 0$ will hold in the optimal harvest programme and the recruitment function is then concave in the number of females.[11] Table 1 gives the baseline parameter values for this function as well as the economic parameter values. In the present exposition we want to mimic the management of the whole Norwegian moose population. The baseline value of the parameter K is scaled in this way and is assumed to be 80,000 animals (females). Notice also that it is assumed a zero discount rate in the baseline scenario. It is well known that the steady state of the dynamic optimization problem then coincides with the problem of maximizing the current animal benefit in biological equilibrium. Hence, the steady state results are similar with the results of the equilibrium harvest problem.

Table 1 about here

6.2 Results

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[11] Differentiation yields $\partial X_{c,t}/\partial X_{f,t} = [K^b + (1-b)X_{f,t}{}^b](1-e^{-aX_{m,t}})/(K^b + X_{f,t}{}^b]^2$. Moreover, we find $\partial^2 X_{c,t}/\partial X_{f,t}{}^2 = -v[(b+1)K^b + (1-b)X_{f,t}{}^b]$ where the coefficient v collects terms and is positive. It is easily recognized that $\partial X_{c,t}/\partial X_{f,t} \geq 0$ implies $\partial^2 X_{c,t}/\partial X_{f,t}{}^2 \leq 0$. For the given parameter values of b and K (Table 1), we find $\partial X_{c,t}/\partial X_{f,t} \geq 0$ when $X_{f,t} \leq 80,000$.

We first present the basic dynamic harvesting results.[12] As already mentioned (section four) it is difficult, if not impossible, to say something analytically about the dynamics. However, due to the strong degree of linearity in the model together with density dependent regulating through the recruitment function, we may suspect the model to approach a stable equilibrium. This is confirmed by the numerical results where the dynamics have similarities with the Most Rapid Approach Path (MRAP), but with some degree of undershooting and overshooting depending on the initial situation, parameter values and management regime. Figure 4 shows the harvest rate dynamics with the baseline parameter values under the overall management scenario (OM). Under this management scheme and for these parameter values and initial values, assumed to be 30,000 for all categories of animals, there is no harvest of yearlings, but harvest of calves and females, together with males. As seen, the harvest rates seem to reach the equilibrium, or steady state, values quite fast. The robustness of the dynamics is examined by changing some of the parameters of the model. We find that a positive discount rent (5 %), not surprisingly, yields somewhat higher harvest rates and lower total stock value while the dynamics does not change qualitatively. The effects of other initial stock values were examined as well without changing the qualitative structure of the dynamics; neither was other steady – state values found (‘ergodic’ dynamics). These and other dynamic sensitivity results are available from the authors upon request.

Figure 4 about here

Tables 2 and 3 report the detailed steady state results under the baseline parameter value scenario for all management regimes. In addition, we have included recent harvest and stock data in Norway where the present total stock is about 160,000 and total harvest is about 36,000 individuals, indicated by the harvesting scheme denoted as Current (see also Figure 1). In the LO regime with meat value maximization only, the calf, yearling, female and male harvest rates become 0.00, 0.81, 0.00 and 0.14, respectively. The low male harvest rate may seem surprising as this is the far most valuable animal category (cf. Table 1). The analytical results (section four) showed

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[12] The dynamic optimization was performed with the Premium Solver Platform from Frontline Systems.

that a positive yearling harvest could not be beneficial with an unbinding female- calf harvest constraint (10). On the other hand, when this constraint binds, we found that yearling harvest together with male harvest could possibly also represent the optimal strategy, which is exactly what happens here. It occurs when the constraint binds with zero female as well as zero calf harvest as discussed above (section four). In this case and for the baseline parameter values it turns out that since the female harvest is restricted, controlling the female population by harvesting yearlings is beneficial. The high yearling harvest rate leaves few males. This comes at the cost of a low male harvest rate, and the whole population is almost completely controlled by yearling harvest alone. Therefore, in this LO regime the yearling control condition (12) (without damage cost) holds as an equation, meaning that the yearling marginal harvest value equalizes the average survival adjusted shadow value of the adult female and male animals. The male control condition holds as an equation as well. On the other hand, the calf control condition (11) indicates that the calf harvest value is below that of the survival adjusted yearling shadow value minus the shadow value of the female – calf harvest constraint.

Note that these results confirm that $\partial X_{c,t} / \partial X_{f,t} = (r + r'_f X_{f,t}) > 0$ holds, and hence the female population size is below the peak value of the recruitment function, i.e., 80,000 animals for the given parameter values (footnote 11). At the same time, this means that this steady state solution is located at the upward bending part of the male population equilibrium condition in the above Figure 3. This will also hold in all regimes (see below). The harvest pattern in the LO regime means that that the adult female population becomes far higher than the male population, and the adult population proportion X_m / X_f yields 0.28 (20,860/74,520). As shown above (section three), only the adult harvest influences this proportion. The annual meat value (Q) is about NOK 372 million (Table 3). Adjusted for browsing and traffic damage costs, the annual overall surplus ($Q - D - T$) reduces to about NOK 58 million in this LO scheme.

Tables 2 and 3 about here

In the LOF regime where the browsing damage cost is included, the harvest, as suspected, become generally higher than under the LO scheme. Male harvest is now accompanied by female and calf harvest while the yearling population is unexploited. The calf, female and male harvest rate increases from 0.00 to 0.26, from 0.00 to 0.17 and from 0.14 to 0.80, respectively, while the yearling harvest rate decreases from 0.81 to 0.00. Hence, a substantial change in the harvest pattern takes place when the browsing damage cost is included. The adult population proportion X_m / X_f reduces moderately to 0.27 as a higher male harvest rate is accompanied by a higher female rate as well (again, see section three). Overall, the total stock decreases from 182,000 animals to 157,500, where the bulk of the reduction comes from lower yearling and female abundance. The reduction in the yearling stock, even though there is no harvest of this category, is due to the direct effect of increased calf harvest and the indirect effect of increased female harvest. The relatively small reduction of the total stock (13.5%) leads to an even smaller reduction in the harvest income; from about 372 to about 362 million (<3%). This may be explained by the fact that as more calves are harvested, the weight loss of harvesting calves instead of yearlings is more than compensated by the increase in male and female harvest. At the same time, as the composition of the stock is altered towards less males and females, the forests as well as the traffic damage cost are reduced. In the end, the yearly overall surplus $(Q - D - T)$ increases by about 85% (from about 58 to 107 million) when the browsing damage 'externality' is taken into account (cf. section two above). Note also that by taking the forest browsing damage into account, the landowners, as a by-product, reduces the traffic damage cost and thereby moves the solution substantially closer to the overall management solution.

As discussed above (section four), the harvest rate of females (0.24), males (0.94) and calves (0.27) are higher in the overall management scenario OM than under any of the other regimes while the yearling harvest is zero. The yearling population is dramatically reduced; now from about 41,000 individuals under the LO scheme to somewhat below 26,000 under OM. This reduction is again due to increased harvest of calves and females as explained above. Note that almost all adult males are harvested under the OM regime, and hence only those needed to secure recruitment are spared. However, note also that the male yearling entering the adult stage takes part in reproduction from the male side. This male harvest rate may seem extreme, but hinges crucially on the magnitude of the traffic damage costs. The other age classes are also reduced significantly, leaving the total stock at somewhat above 126,000 animals. The yearly hunting income is reduced to about NOK 322 million while the overall surplus is 130% higher than under the LO regime, and about 23% higher than in the LOF regime. Compared to the current management regime discussed below, the overall management would increase the overall surplus by about 30%; from about 101 million to 132 million NOK.

As indicated, Current reflects the present harvest pattern in Norway, and is based on the official hunting statistics (SSB 2009) together with the Table 1 cost and price assumptions. Here, all stages are harvested and the harvest rates of calves, yearlings, females and males are 0.19, 0.28, 0.13 and 0.60, respectively. This harvest scheme indicates that both the traffic damage cost (T), the browsing damage cost (D) as well as hunting income (Q) are somewhat between the LO and LOF results. Furthermore, it seems evident that the results are closer to the LOF than the LO regime and the overall surplus is less than 5% below the outcome of the LOF regime. These results demonstrate the harvest trade-offs between the different stages as the composition of harvest between stages can be quite different. Hence, even though the Current harvest scheme is quite different from the LOF, the steady state population levels and total harvest become quite similar. For example, yearling harvest is zero under LOF and 0.28 under Current while the overall harvest and profit change just modestly. This clearly indicates that different harvest composition is able to produce quite similar population levels. The intuition is that even if calves and yearlings may be seen as substitutes in reducing the adult stocks, it turns out by coincidence that the economic loss of exploiting both these stages becomes quite moderate in this specific case. Hence, not harvesting the yearling stage can be compensated by harvesting more of the calves.

6.3 Sensitivity analysis and model extension

We have also assessed how sensitive the steady state results are to the cost assumptions. In the following, 10% increases in the baseline browsing and traffic damage cost parameters are considered while all other parameters are kept unchanged.

Table 4 indicates that these changes have quite modest harvest as well as stock effects. For example, the 10% cost increase reduces the overall stock size under the OM scheme by less than 4%, from about 126,000 to about 122,000 (Tables 2 and 4). However, the economic consequences are more severe as the overall yearly surplus ($Q-D-T$) is reduced by about 14% , from about 132,000 to about 113,500 (Tables 3 and 5). This indicates that the profit function is strictly nonlinear. Obviously, the overall surplus difference between the LO and the OM regimes increases when the cost increases since the LO regime is based on maximizing hunting value only. The overall surplus difference between the OM and the LOF regime increases as well because the stock is adjusted only with respect to the browsing damage under the LOF scheme. On the other hand, as the economic differences between the management regimes increase due to higher costs or that more cost components are included, the economic differences between the regimes increases as well. In the opposite case with both zero traffic and browsing costs, the overall surplus will of course be identical under all schemes.

Tables 4 and 5 about here

Just as in Ericsson et al. (2000), we have also calculated the net benefit when our optimal selective harvesting pattern is replaced by an ‘optimal’ uniform hunting pattern. This uniform hunting pattern describes an exploitation scheme where the present value net benefit is maximized under the biological constraints together with the female – calf restriction (10) and the condition of similar harvest rates among *all* stages. This scheme may thus indicate a harvest pattern where ‘an animal is an animal’ as considered in biomass models used in the traditional bioeconomic analysis (e.g., Clark 1990). The ‘optimal’ uniform steady state harvest rate becomes 0.15, 0.18 and 0.22 in the LO, LOF, and OM regime respectively. See Table 6. The economic benefits reduce dramatically (results available from the authors upon request).

Table 6 about here

The cost of the female – calf restriction (10) was also calculated. Without this restriction, only adult harvest is optimal (section four) and both stages are hunted. We find that the net benefit in the LOF regime increases from about NOK 107,000 (Table 3) to NOK 164,500. Hence, the code of conduct harvest restriction comes at a considerable cost.

In our model, linear benefit and cost functions have been assumed. These assumptions may be crucial for the results, and we have therefore studied the optimal harvest pattern when stock dependent harvest costs are included. The results are shown in the Appendix where we also demonstrate how the first order necessary conditions changes. The main numerical finding is that the harvest *pattern* is only modestly affected when the unit harvest cost is assumed similar for all stages. Hence, the exploitation of yearlings remains zero in the LOF regime for the wide range of stock dependent costs we study, while the harvest rates for females and calves change only marginally. However, as we increases the harvest cost, still assumed to be similar for all stages, the harvest rate for males decreases, just as expected (Table A1, Appendix).

Finally, a numerical analysis is also included when the linear damage cost functions assumption is replaced by convex functions. In the LOF regime, we find quite similar harvest pattern as in the linear case (Table A2 Appendix).

7. Concluding remarks

In this paper we have analysed the cost and benefit of the Scandinavian moose population within a four stage model with density-dependent fertility and density independent mortality and where the cost and benefit functions are approximated by linear functions. Two basic exploitation schemes, landowner exploitation (LO and LOF) and overall management (OM), have been studied. The different ways to compose the harvest, e.g. in yearlings or females, and how the various management regimes induce different composition of the harvest are highlighted. Without a restriction on the female – calf harvest, we find the optimal harvest composition to be determined basically by the same factors as in Reed (1980). With the female – calf restriction included, typically neglected in the existing literature on moose harvesting, we find it to bind. As a consequence, the harvest composition will be substantially different from the situation without this constraint. Therefore, the same number of calves and females should always be harvested (zero or positive) in the optimal solution, irrespective of harvesting regime.

The numerical section illustrates the predictions from the theoretical model. In the hunting value only management scheme (LO-scenario), we find that zero calf and female harvest and high yearling harvest are accompanied by a modest male harvest. We also find that under the overall management scenario (OM), no yearling harvest is optimal while calves and females should be harvested. Moreover, the male stage is more aggressively harvested than the other stages in the LOF and OM regime. It is also demonstrated how changing harvest mortality of the different stages is accompanied by significant profitability changes while leaving total harvest more or less unchanged.

Comparing the current management regime of moose in Norway (Current) with the overall management (OM) regime studied here shows that the moose stock in Norway generally is far too high. The calculated yearly overall loss is about NOK 30 million. The most significant difference between the two solutions is that the OM management regime suggests that no yearlings should be harvested, while the harvest of yearlings in Current is substantial. Therefore, from an overall perspective, this analysis indicates that the moose stock in Norway is too high causing too much browsing and traffic damage compared to the hunting value income obtained. The massive increase in the moose stock in Scandinavia since the seventies must be seen as a large scale ecological project to maximize meat production. From an economic point of view, maximizing meat production without taking forest and traffic damage costs into account seems strange. Thus, a harvest pattern of the different age classes that maximizes meat production is not in accordance with the economic optimal harvest pattern.

Our study is a restricted type of cost-benefit analysis because some values, like non-use values and existence values, are neglected. However, due to the recent high moose population in Scandinavia, such values, on the margin, are probably quite modest and will hence only have small influence on the optimal harvest composition. All our basic conclusions drawn about the harvesting composition is based on linear damage cost and benefit functions. However, when relaxing the linearity assumptions, numerical results indicate that the harvest pattern does not change dramatically. We have also calculated the net benefit when our optimal selective harvesting pattern is replaced by an ‘optimal’ uniform pattern where an ‘animal is an animal’ as considered in the traditional bioeconomic models. We find that such harvesting pattern reduce the economic benefit considerably. Finally, while the present analysis is for a moose population, our results have certainly relevance for the management of other species, cf. the Reed (1980) paper as well as the recent age structured fishery analysis in Tahvonen (2009).

Appendix

Stock dependent harvest costs

In this Appendix, we first examine how density dependent hunting costs may influence the harvesting decision. We may think of this as the situation where the landowners hunt by themselves with search costs, but no recreation value, and where these costs depend on the moose density. We use the standard Schaefer harvesting function approach under the assumption of perfect selectivity $H_i = q_i E_i X_i$ $(i = c, y, f, m)$ with E_i as effort (measured in number of hunting days) and q_i as the catchability coefficient (1/hunting day), generally assumed to be different for the different categories of animals. When v is the unit effort cost (NOK/hunting day), the current net harvesting benefit reads:

$$Q_t = [pw_c h_{c,t} r(X_{f,t}, X_{m,t}) X_{f,t} - \left(v / q_c\right) h_c + pw_y h_{y,t} X_{y,t} - \left(v / q_y\right) h_y$$
$$+ pw_f h_{f,t} X_{f,t} - \left(v / q_f\right) h_f + pw_m h_{m,t} X_{m,t} - \left(v / q_m\right) h_m].$$

It can easily be confirmed that the first order control conditions under the LOF scenario may be written as:

$$\frac{\partial L}{\partial h_{c,t}} = rX_{f,t}(pw_c + d_c - \frac{v}{q_c r X_{f,t}} - \rho\eta_{t+1} s_c + \rho\omega_{t+1}) \le 0 \;;\; 0 \le h_{c,t} < 1, \tag{11`}$$

$$\frac{\partial L}{\partial h_{y,t}} = X_{y,t}(pw_y + d_y - \frac{v}{q_y X_{y,t}} - \rho\lambda_{t+1} 0.5 s_y - \rho\mu_{t+1} 0.5 s_y) \le 0 \;;\; 0 \le h_{y,t} < 1, \tag{12`}$$

$$\frac{\partial L}{\partial h_{f,t}} = X_{f,t}(pw_f + d_f - \frac{v}{q_f X_{f,t}} - \rho\lambda_{t+1} s - \rho\omega_{t+1}) \le 0;\; 0 \le h_{f,t} < 1 \tag{13`}$$

and

(14`) $$\frac{\partial L}{\partial h_{m,t}} = X_{m,t}(pw_m + d_m - \frac{v}{q_m X_{m,t}} - \rho\mu_{t+1}s) \leq 0 \quad ; 0 \leq h_{m,t} < 1 .$$

The state, or portfolio conditions, with $X_{c,t} > 0$, $X_{y,t} > 0$, $X_{f,t} > 0$ and $X_{m,t} > 0$, will be similar to what we found without stock dependent costs (Equations 15 – 17).

Even when the female – calf harvesting restriction does not bind, the analysis of the harvesting composition now becomes quite complex as the stock sizes are included in the control conditions. This analysis is left out for future research. However, we have examined some numerical effects where we assume identical catchability coefficients for all stages, $q_c = q_y = q_f = q_m = 0.00000041$. The size of the catchability coefficients are calibrated based on an assumption that 60,000 hunters hunt approximately 10 days per year on average (Statistics Norway 2010). With a total stock of 150,000 moose, these catchability coefficients correspond to a total harvest of 36,900 moose per year (see also Table 2). The numerical effects are studied under various cost assumptions in the LOF regime. See Table A1.

Table A1 about here

We have also examined the case where it is easier to shoot calves and yearlings than the mature female and male moose; that is, the catchability coefficients for these categories become higher than that of the adults. We now find yearling harvest to be profitable for a certain range of the effort cost parameter (results are available upon request). It seems reasonable to disentangle this switching harvest pattern as a consequence of the more efficient harvest of the yearling stage. However, we are not able to rule out that the stock dependent harvesting costs also may play a role.

Convex damage cost functions

Strictly convex damage costs are introduced as:

(9`)
$$D_t = d_c(1-h_{c,t})X_{c,t} + \frac{\sigma_c}{2}\left[(1-h_{c,t})X_{c,t}\right]^2 + d_y(1-h_{y,t})X_{y,t} + \frac{\sigma_y}{2}\left[(1-h_{y,t})X_{y,t}\right]^2$$
$$+ d_f(1-h_{f,t})X_{f,t} + \frac{\sigma_f}{2}\left[(1-h_{f,t})X_{f,t}\right]^2 + d_m(1-h_{m,t})X_{m,t} + \frac{\sigma_m}{2}\left[(1-h_{m,t})X_{m,t}\right]^2$$

with $\sigma_i > 0\,(i = c, y, f, m)$. The harvest and stock composition results for different values of σ_i are shown for the LOF regime in Table A2 where the main section baseline values of d_i are assumed. Not surprisingly, the stocks reduce compared to the linear damage cost case. On the other hand, the harvest composition changes only slightly (Table 2, main text).

Table A2 about here

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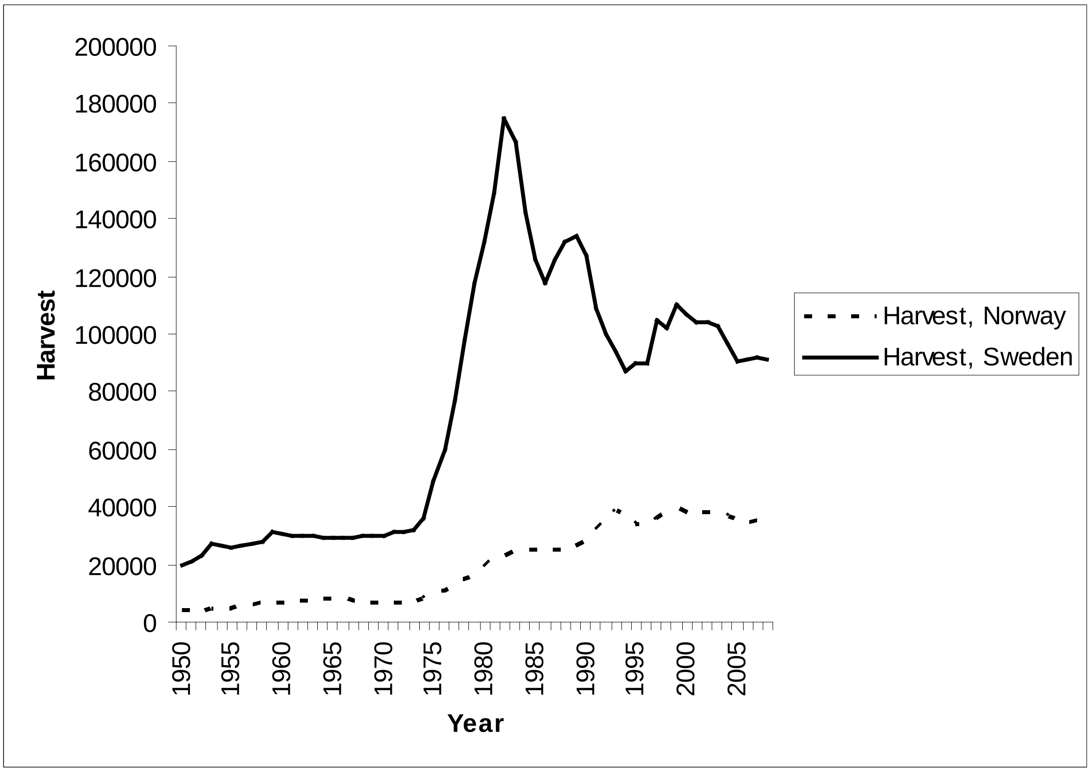

Figure 1: Number of hunted moose Norway and Sweden 1952 -2008. Source: SSB, Statistics Norway (2009) and Svenska Jägerforbundet (2009).

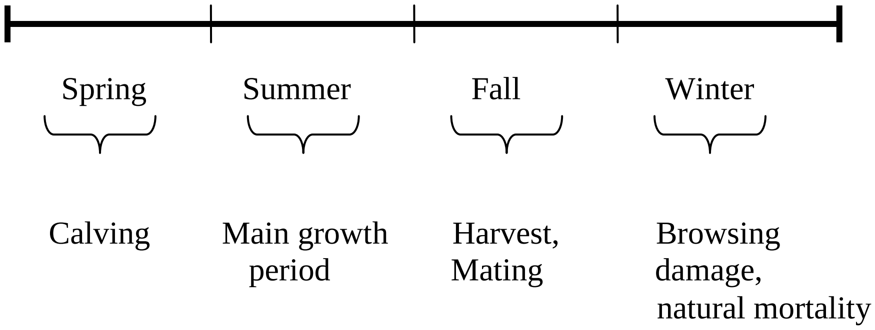

Figure 2: Seasonal pattern and course of major events Scandinavian moose.

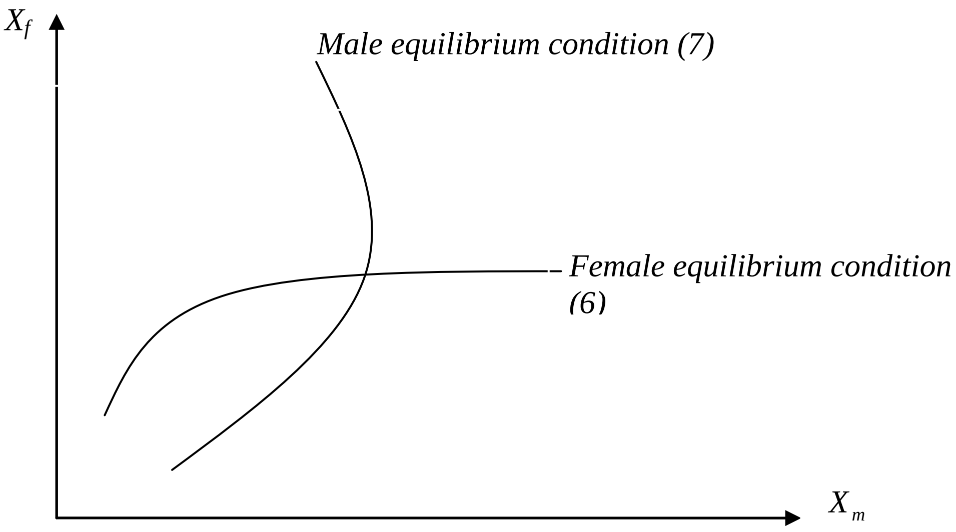

Figure 3: Biological equilibrium male (X_m) and female (X_f) population stages. Fixed harvest rates.

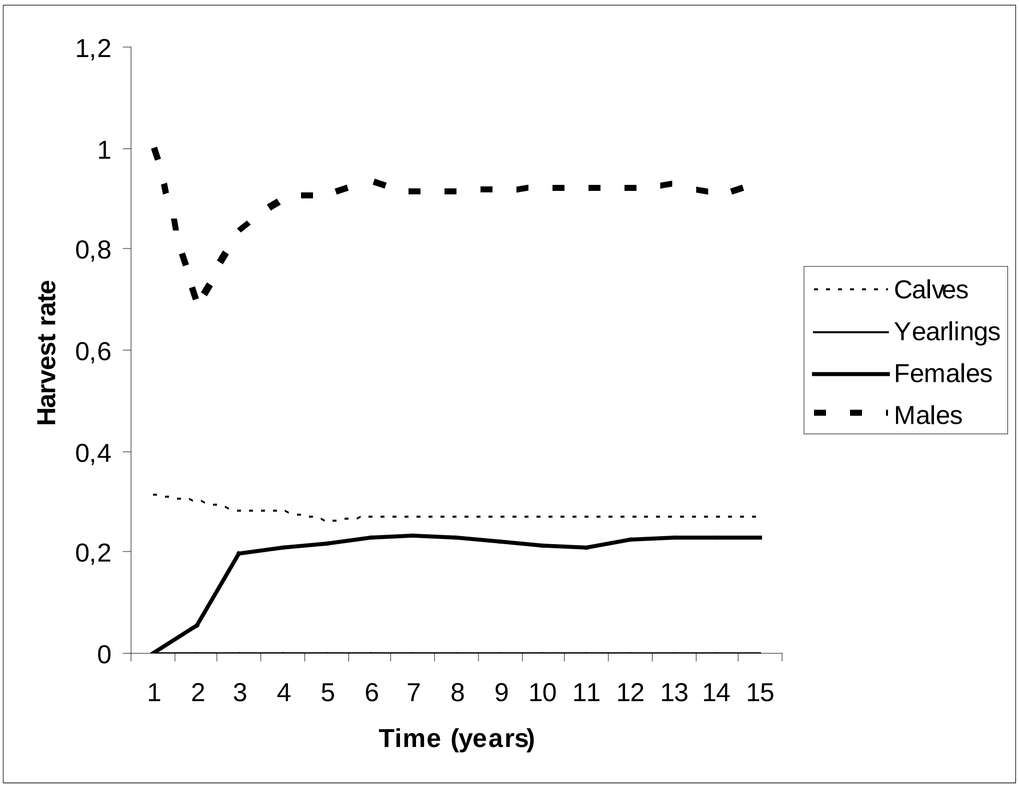

Figure 4: Harvest rates overall management scenario (OM). Baseline parameter values. Yearlings harvest rate all the time zero.

***Table 1:** Baseline biological and economic parameter values*

Parameters	Description	Value	Reference/source
$\tilde{r}$	maximum specific growth rate	1.15	Nilsen et al. (2005),
K	female stock level where density dependent factors dominate density independent factors	80,000 animal	Calibrated
a	male density recruitment factor	0.00024	Calibrated
b	density compensation parameter	2	Nilsen et al. (2005)
w_c	average weight calve	65 kg/animal	SSB (2004)
w_y	average weight young	135 kg/animal	SSB (2004)
w_f	average weight female	150 kg/animal	SSB (2004)
w_m	average weight male	170 kg/animal	SSB (2004)
s_c	natural survival rate calf	0.90	Nilsen et al. (2005)
s_y	natural survival rate young	0.95	Nilsen et al. (2005)
s	natural survival rate female and male	0.95	Nilsen et al. (2005)
p	meat price	75 NOK/kg	Storaas et al. (2001)
d_c	marginal browsing damage calf	250 NOK/calf	Larsen (2007)
d_y	marginal browsing cost yearling	500 NOK/yearling	Larsen (2007)
d_f	marginal browsing cost female	750 NOK/female	Larsen (2007)
d_m	marginal browsing cost male	750 NOK/male	Larsen (2007)
t_c	marginal traffic cost calf	800 NOK/calf	Solstad (2007)
t_y	marginal traffic cost yearling	1,700NOK/yearling	Solstad (2007)
t_f	marginal traffic cost female	1,900 NOK/female	Solstad (2007)
t_m	marginal traffic cost male	2,100 NOK/male	Solstad (2007)
δ	discount rate	0	

Table 2: *Steady state different management regimes, baseline parameter values. LO landowner exploitation, LOF landowner exploitation taking forest browsing damage into account and OM overall management taking both browsing costs and traffic damage costs into account. Current reflects actual harvesting data.* X_c, *number of calves,* X_y *number of yearlings,* X_f *number of adult females,* X_m *number of adult males,* X *total number of animals,* H_c *number of calf harvest,* H_y *number of yearling harvest,* H_f *number of female harvest,* H_m *numner of male harvest and* H *total harvest. Corresponding harvest rates in brackets.*

	X_c	X_y	X_f	X_m	X	H_c	H_y	H_f	H_m	H
LO	45,580	41,020	74,520	20,860	181,980	0 ($h_c=0$)	33,190 ($h_y=0.81$)	0 ($h_f=0$)	2,816 ($h_m=0.14$)	36,006
LOF	44,450	29,740	65,790	17,480	157,460	11,420 ($h_c=0.26$)	0 ($h_y=0$)	11,380 ($h_f=0.17$)	13,950 ($h_m=0.80$)	36,750
Current	44,510	32,620	65,660	17,960	160,750	8,279 ($h_c=0.19$)	9,166 ($h_y=0.28$)	8,273 ($h_f=0.13$)	10,780 ($h_m=0.60$)	36,498
OM	39,010	25,800	48,530	13,040	126,380	10,340 ($h_c=0.27$)	0 ($h_y=0$)	10,340 ($h_f=0.24$)	12,210 ($h_m=0.94$)	32,890

Table note: H_c may differ slightly from H_f due to rounding errors.

Table 3: *Steady state different management regimes, baseline parameter values. LO landowner exploitation, LOF landowner exploitation taking browsing damages into account and OM overall management taking both forest browsing and traffic damage costs into account. Current reflects actual harvesting data. D is total forest browsing damage cost, T is total traffic damage cost and Q is hunting value. All values in thousand yearly NOK.*

	D	*T*	*Q*	*(Q – D – T)*
LO	84,740	229,300	371,900	57,860
LOF	66,580	187,800	361,600	107,220
Current	69,210	193,000	363,600	101,390
OM	49,340	141,100	322,300	131,860

***Table 4**: Steady state different management regimes, 10% increase of traffic and damage costs compared to baseline parameter values. LO landowner exploitation, LOF landowner exploitation taking browsing damages into account, Current reflects actual harvesting data, OM overall management taking both browsing costs and traffic damage costs into account.* X_c, *number of calves,* X_y *number of yearlings,* X_f *number of females,* X_m *number of males,* X *total number of moose,* H_c *harvest of calves,* H_y *harvest of young,* H_f *harvest of females,* H_m *harvest of males, and* H *total harvest. Corresponding harvest rates in brackets.*

Table note: H_c may differ slightly from H_f due to rounding errors.

	X_c	X_y	X_f	X_m	X	H_c	H_y	H_f	H_m	H
LO	45,580	41,020	74,520	20,860	181,980	0 (h_c=0)	33,190 (h_y=0.81)	0 (h_f=0)	2,816 (h_m=0.14)	36,006
LOF	44,310	29,630	65,010	17,250	156,200	11,390 (h_c=026)	0 (h_y=0)	11,380 (h_f=0.18)	13,900 (h_m=0.81)	36,670
Current	44,510	17,960	65,660	32,620	160,750	8,279 (h_c=0.19)	9,166 (h_y=0.28)	8,273 (h_f=0.13)	10,780 (h_m=0.60)	36,498
OM	38,000	25,110	46,430	12,610	122,140	10,120 (h_c=0.27)	0 (h_y=0)	10,110 (h_f=0.22)	11,890 (h_m=0.94)	32,120

Table 5: *Steady state different management regimes, 10% increase of traffic and damage costs compared to baseline parameter values. LO landowner exploitation, LOF landowner exploitation taking browsing damages into account, Current reflects actual harvesting data, OM overall management taking both browsing costs and traffic damage costs into account. D is total forest browsing damage costs, T is total traffic damage costs, Q is meat value, OS is overall surplus taking hunting income, traffic damage and browsing damage costs into account. All values in thousand yearly NOK.*

	D	*T*	*Q*	*(Q – D – T)*
LO	93,210	252,200	371,900	26,490
LOF	72,360	204,200	360,800	84,240
Current	76,130	212,300	363,600	75,170
OM	52,030	149,100	314,800	113,670

Table 6: *Steady state different management regimes, baseline parameter values uniform harvest rates. LO landowner exploitation, LOF landowner exploitation taking forest browsing damage into account and OM overall management taking both browsing costs and traffic damage costs into account.* X_c, *number of calves,* X_y *number of yearlings,* X_f *number of adult females,* X_m *number of adult males,* X *total number of animals,* H_c *number of calf harvest,* H_y *number of yearling harvest,* H_f *number of female harvest,* H_m *number of male harvest and* H *total harvest. Corresponding harvest rates in brackets.*

	X_c	X_y	X_f	X_m	X	H_c	H_y	H_f	H_m	H
LO	45,700	34,810	71,360	71,360	232,200	7,040 (0.15)	5,360 (0.15)	10,990 (0.15)	10,990 (0.15)	34,380 (0.15)
LOF	43,570	32,230	57,390	57,390	190,600	7,760 (0.18)	5,740 (0.18)	10,220 (0.18)	10,220 (0.18)	33,940
OM	29,480	20,610	29,040	29,040	108,200	6,570 (0.22)	5,000 (0.22)	6,480 (0.22)	6,480 (0.22)	24,530

Table A1: *Steady state under different stock dependent harvest costs, LOF landowner exploitation taking forest browsing damage into account* $X_{c,}$ *number of calves,* X_y *number of yearlings,* X_f *number of adult females,* X_m *number of adult males, X total number of animals,* H_c *number of calf harvest,* H_y *number of yearling harvest,* H_f *number of female harvest,* H_m *number of male harvest and H total harvest. Corresponding harvest rates in brackets.*

	X_c	X_y	X_f	X_m	X	H_c	H_y	H_f	H_m	H
LOF	44,450	29,740	65,790	17,480	157,460	11,420 (h_c=0.26)	0 (h_y=0)	11,380 (h_f=0.17)	13,950 (h_m=0.80)	36,750
LOF*	44,860	30,000	66,010	20,830	161,700	11,530 (h_c=0.26)	0 (h_y=0)	11,550 (h_f=0.18)	13,910 (h_m=0.67)	36,990
LOF**	45,410	30,420	68,210	35,810	179,850	11,620 (h_c=0.26)	0 (h_y=0)	11,600 (h_f=0.17)	13,320 (h_m=0.37)	36,540
LOF***	45,760	30,770	72,290	50,690	199,510	11,580 (h_c=0.25)	0 (h_y=0)	11,570 (h_f=0.16)	12,720 (h_m=0.25)	35,870
LOF****	45,970	31,060	77,380	62,370	216,780	11,450 (h_c=0.25)	0 (h_y=0)	11,450 (h_f=0.15)	12,220 (h_f=0.20)	35,120

Table note: H_c may differ slightly from H_f due to rounding errors.
LOF: Baseline with zero stock dependent harvest costs (cf. main text Table 2).
LOF*: Stock dependent unit harvest costs $v = 10$ (NOK/hunting day)
LOF**: Stock dependent unit harvest costs $v = 50$
LOF***: Stock dependent unit harvest costs $v = 100$
LOF****: Stock dependent unit harvest costs $v = 150$

Table A2: *Steady state, convex browsing damage cost function. LOF landowner exploitation taking forest browsing damage into account.* X_c, *number of calves,* X_y *number of yearlings,* X_f *number of adult females,* X_m *number of adult males, X total number of animals,* H_c *number of calf harvest,* H_y *number of yearling harvest,* H_f *number of female harvest,* H_m *number of male harvest and H total harvest. Corresponding harvest rates in brackets.*

	X_c	X_y	X_f	X_m	X	H_c	H_y	H_f	H_m	H
LOF*	39,320	25,940	47,060	15,880	128,200	10,500 (0.27)	0 (0)	10,490 (0.22)	12,130 (0.76)	32.670
LOF**	33,550	22,010	36,410	14,230	106,200	9,090 (0.27)	0 (0)	9,100 (0.25)	10,260 (0.72)	28,450
LOF***	38,200	25,160	44,460	15,860	123,700	10,240 (0.27)	0 (0)	10,230 (0.23)	11,740 (0.74)	32,210

Table note: H_c differs slightly from H_f due to rounding errors.

* $\sigma_c = \sigma_y = \sigma_f = \sigma_m = 0.05$, ** $\sigma_c = \sigma_y = \sigma_f = \sigma_m = 0.1$,

*** $\sigma_c = 0.025,\ \sigma_y = 0.05,\ \sigma_f = \sigma_m = 0.075$.