Journal of Environmental Economics and Management 92 (2018) 125–133

Contents lists available at ScienceDirect

Journal of Environmental Economics and Management

journal homepage: www.elsevier.com/locate/jeem

New insights from the canonical fisheries model – Optimal management when stocks are low

Eric Nævdal [a,*], Anders Skonhoft [b]

[a] Frisch Centre, University of Oslo, Gaustadallén 21, N-0371, Oslo, Norway
[b] Department of Economics, Norwegian University of Science and Technology, Norway

ARTICLE INFO

Article history:
Received 5 July 2018
Received in revised form 8 September 2018
Accepted 14 September 2018
Available online 20 September 2018

jel codes:
C61
Q22

ABSTRACT

We analyse the standard optimal control fishery biomass model and derive some novel results on optimal management when fish stocks are low. We show that as long as it is not optimal to let the stock become extinct and the marginal benefit of harvesting is bounded below infinity for all harvest levels, there will always be an interval with low stock sizes where it is optimal not to harvest. This result does not depend on any assumption that marginal harvesting cost per unit increases with decreasing stock size. We then prove that under weak conditions the shadow price on the fish stock always goes to infinity as the stock approaches zero. The results are generalized to a particular class of age structured models.

1. Introduction

Clark (1973) and Clark and Munro (1975) presented dynamic fishery models that gave the theory of renewable resources a proper capital theoretic foundation. The basic fishery model entails one control variable, one state variable; the planning horizon is infinite time and the problem is autonomous. When the profit function is nonlinear in the control variable and there is an optimal path to the steady state, this steady state should be approached gradually along two saddle paths, or stable manifolds (see, e.g., Kamien and Schwarz, 1991). The standard model has usually applied an ecological lumped parameter model of the form $\dot{x} = G(x) - h$ where x is the size of the fish stock in biomass and h is the harvest rate. It has been recognized for a long time that optimal extinction in these models depends on the relative magnitude of the interest rate and the intrinsic growth rate, $G'(0)$, in addition to the unit cost of harvesting (Clark, 1973; Cropper et al., 1979). Although this model is well understood, some wrinkles remain to be ironed out. One is the question of harvest levels at low stock levels, where it is has been known that in some versions of the standard fisheries model it is optimal to set harvest equal to zero for low stock levels. This is commonly attributed to either the bang-bang nature of problems that are linear in the control, Clark and Munro (1975) or an assumption that harvest costs are stock dependent and that the marginal cost of harvest becomes infinite when the stock approaches zero, (Leung and Wang, 1976; Lewis and Schmalensee, 1977).[1]

* Corresponding author.
E-mail address: eric.navdal@frisch.uio.no (E. Nævdal).

[1] On the other hand, if the marginal benefit of harvesting goes to infinity as the harvest rate goes to zero, typically in models with some type of iso-elastic instantaneous utility, then if stocks are strictly positive it is always optimal with some strictly positive harvest rate, Levhari and Mirman (1980). Whether extinguishing the fish stock is optimal will also in this case depend on the relationship between the discount rate and the intrinsic growth rate.

https://doi.org/10.1016/j.jeem.2018.09.003

In what follows, we show that these assumptions are not necessary. In order to properly analyse optimal harvest levels at low stocks, it is crucial to examine the behaviour of the shadow price at low stock levels. We argue below that analysing the properties of the shadow price is equivalent to analysing the stable saddle path in a phase diagram in stock/shadow price space. If we interpret the stable saddle path as a function that maps the state variable into the shadow price it is evident that the stable saddle path is in fact the derivative of the value function. We then demonstrate that the shadow price of a renewable resource goes to infinity if the growth in the resource is zero at zero stock. This fact has remarkably not been noted in the literature, except for the case where revenue is a linear function of harvest levels, Nævdal (2016). In his milestone book on natural resource economics Colin Clark stays silent on this. He draws the basic fishery model phase-diagram in the stock – harvest space, but the saddle path is not drawn for low harvest levels, Clark (1991, p. 99) and also Conrad and Clark (1987, p. 56). In the well-recognized book by Leonard and Long (1992) on optimization and dynamic control models, the saddle path illustrating a schooling fishery is only indicated for a restricted set of values in the stock – shadow price space (Leonard and Long (1992, p. 296) and is not drawn for values of the stock close to zero.

In Section 2 below, we first formulate and analyse our baseline model exemplified by a schooling fishery where the net harvest benefit is a concave function of harvest. In Section 3, we next apply fast/slow-dynamics and show that the results apply to at least some age structured models. Section 4 concludes the paper with a discussion of the results and relating them to the concept of harvest control rules.

2. The canonical fisheries model

The following is the basic version of the fisheries model where a schooling fishery is considered. In a schooling fishery there are no stock dependent harvest costs. We assume that the net instantaneous benefits from harvesting is given by a continuous and strictly concave function $D(h)$ with $D(0) = 0$, and where $D'(h) > 0$ over an interval $[0, h_{max}]$ where $h_{\max} \leq \infty$. For notational convenience we denote $D'(h)$ as $d(h)$. Note that strict concavity of $D(h)$ ensures that $d(h)$ has an inverse defined for all positive values of its argument. In order to ensure that our results are not the result of assuming infinite derivatives of $D(h)$, we postulate that $0 < d(0) < \infty$ which is a crucial assumption driving our results. The natural growth function $G(x)$ is taken to be strictly concave and satisfy $G(0) = 0$, $G'(x) > 0$ over some interval $[0, \bar{x}\ )$ and $G'(x) < 0$ for $x > \bar{x}$. We assume that the intrinsic growth rate exceeds that of the discount rate, $G'(0) > \rho$, which is reasonable for most fish species It is also assumed that there is some number $K > \bar{x}$, denoted carrying capacity, such that $G(K) = 0$. The specification of $G(x)$ is in line with standard growth functions such as the logistic one, which is used in our numerical illustrations. The assumptions lead to the following optimization problem:

$$V(x(0)) = \max_{h \geq 0} \int_0^\infty D(h) e^{-\rho t} dt \quad \text{subject to } \dot{x} = G(x) - h, \text{ and } x(0) \text{ given}, \tag{1}$$

where $h \geq 0$ is the harvest and $x \geq 0$ is the size of the fish stock and $\rho \geq 0$ is the discount rate. The current value Hamiltonian for this problem is:

$$H = D(h) + \mu(G(x) - h). \tag{2}$$

Here μ is the co-state variable. The Hamiltonian is concave in (h, x), so sufficiency theorems such as Theorem 9.11.1 in Sydsæter et al. (2005) are fulfilled. The necessary conditions become:

$$\frac{\partial H}{\partial h} = d(h) - \mu \leq 0 \ (= 0 \text{ if } h > 0) \tag{3}$$

and

$$\dot{\mu} = (\rho - G'(x))\mu. \tag{4}$$

(3) follows from maximising the Hamiltonian with respect to h, when H is a concave function of h. Transversality conditions must also be checked. By assumption there exist a steady state and we show below that the optimal path converges to this steady state from any $x(0) > 0$. It is straightforward to check that $\lim_{t \to \infty} \mu(t)(y(t) - x(t))e^{-\rho t} \geq 0$ where $x(t)$ is the optimal state variable and $y(t)$ all other admissible functions. As $y(t)$, $x(t)$ and $\mu(t)$ are all finite, this expression goes to zero. The transversality condition given in Theorem 9.11.1 in Sydsæter et al. (2005) therefore holds and with the rest of our assumptions implies that sufficient conditions for optimality hold. Control condition (3) implies that $d\ (0) < \mu \Rightarrow h = 0$ and condition (3) may be rewritten as:

$$h = \max\left(0, d^{-1}(\mu)\right). \tag{5}$$

Inserting Eq. (5) into the natural growth equation yields next:

$$\dot{x} = G(x) - \max\left(0, d^{-1}(\mu)\right). \tag{6}$$

We can use Eqs. (4) and (6) to obtain a phase diagram in the (x, μ)-space. The isocline for $\dot{x} = 0$ may be constructed as follows:

$$\begin{gathered} \dot{x} = G(x) - \max\left(0, d^{-1}(\mu)\right) = 0 \\ \Updownarrow \\ \mu = \varphi(x) = d(G(x)) \quad \forall \{x : h > 0\} \end{gathered} \tag{7}$$

Note that $\varphi(0) = \varphi(K) = d(0)$ and that $\varphi(x) < d(0)$ for all $x \in (0, K)$. The isocline for $\dot{\mu} = 0$ is given by:

$$\begin{gathered} \dot{\mu} = (\rho - G'(x))\mu = 0 \\ \Updownarrow \\ \mu = 0 \text{ or } x = G'^{,-1}(\rho) \end{gathered} \tag{8}$$

We shall assume that there is a pair $(x, y) = (x_{ss}, \mu_{ss})$ that solves the equations $\dot{x} = 0$ and $\dot{\mu} = 0$ and hence defines the equilibrium (steady state) of our model. The isoclines in Eqs. (7) and (8) are depicted and discussed in Fig. 1.

To complete the phase diagram we need to draw stable manifolds satisfying the directional derivatives. For $x > x_{ss}$ this is a fairly straightforward task. On the other hand, for $x < x_{ss}$ it is not obvious whether μ along the stable manifold should take values such that $\mu < d(0)$ for all x, or $\mu > d(0)$ if x becomes sufficiently low. We illustrate these two possibilities in Fig. 1 where we draw two hypothetical paths for a stable manifold for x below its steady state value. Note that both these paths satisfy directional derivatives for x and μ both strictly positive. However, there can only be one stable manifold, so we have to choose between them. This is done in Proposition 1, which draws on the observation in Fig. 1 that if there exists a stable saddle path satisfying $\mu < d(0)$ for all $x \in (0, x_{ss})$, then it must originate from the point $(x, \mu) = (0, d(0))$.

Proposition 1. Any path originating from $(x, \mu) = (0, d(0))$ can not be a stable manifold.

Proof. If the stable manifold starts at $(x, \mu) = (0, d(0))$ then its slope is given by:

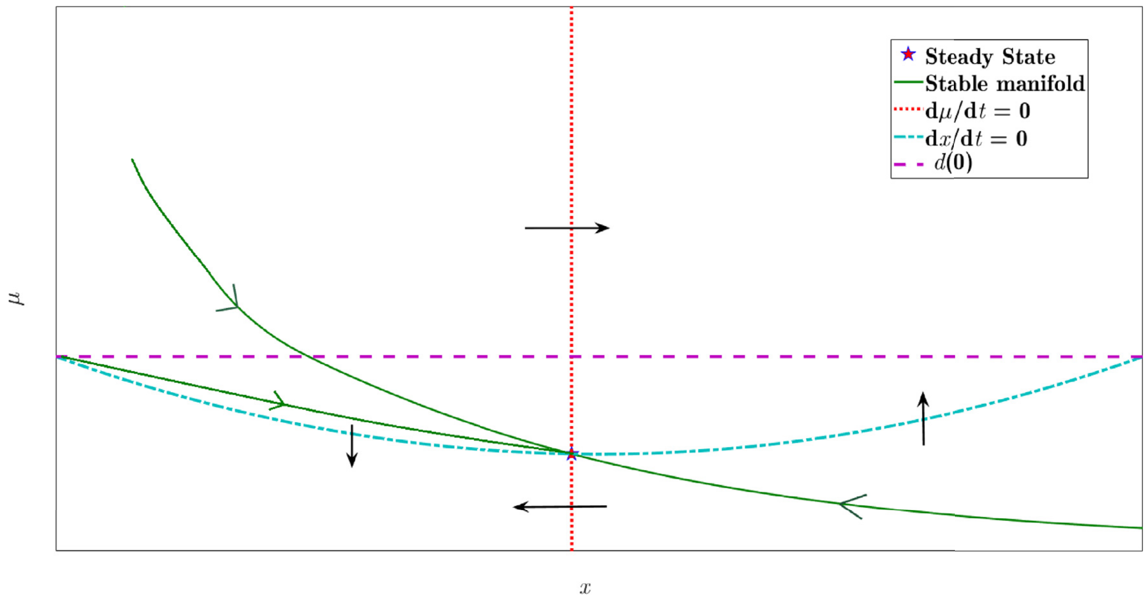

Fig. 1. Isoclines in a phase diagram in the (x, μ)-space. The isoclines at $x = 0$, $x = K$ and $\mu = 0$ are not drawn. The black arrows indicates system directions on the isoclines. The star indicates the steady state point (x_{ss}, μ_{ss}) that solves the equations $dx/dt = 0$ and $d\mu/dt$ and it follows from the directions of the black arrows crossing the isoclines that it is a saddle point as expected. Lines with arrows indicating movement towards the steady state are hypothetical stable saddle paths. For values of x below the steady state there are two paths seemingly satisfying directional derivatives for $x > 0$. One where the stable manifold starts above the $\mu = d(0)$ line, and one where the line lies below the $\mu = d(0)$ line for all $x > 0$. If this last possibility is the case, the stable manifold must start at the point $(x, \mu) = (0, d(0))$. Proposition 1 shows that this is impossible, so the stable manifold must start at some point where $\mu > d(0)$.

$$(d\mu/dx)_{(x,\mu)=(0,d(0))} = (\dot{\mu}/\dot{x})_{(x,\mu)=(0,d(0))} = \frac{d(0)(\rho - G'(0))}{G(0) - \max\left(0, d^{-1}(0)\right)} \\ = \frac{d(0)(\rho - G'(0))}{G(0)} = -\infty \tag{9}$$

This holds under our assumption of an intrinsic growth rate $G'(0)$ that exceeds the rate of discount, and $G(0) = 0$. This slope is clearly smaller than the finite slope of the isocline for $\dot{x}= 0$, so a stable manifold would enter into the area below the isocline for $\dot{x}= 0$, which implies that the stable manifold cannot go through the steady state.

Proposition 1 has a powerful implication that we sum up in a proposition although the Proof is very simple.

Proposition 2. There exists a non-empty interval [0, x^*] where it is optimal to set $h = 0$.

Proof. It follows from Proposition 1 that there exists a stock level x^* where the downward sloping stable manifold crosses the line $\mu = d(0)$, and therefore $h = 0$ for all $x \in [0, x^*]$.

In Nævdal (2016) it was proven that if revenue is linear in harvest, the shadow price would go to infinity as the stock approaches zero. The Proof of this result hinged on the harvest rate being zero if stocks are below the steady state level. Proposition 2 implies that the proof in Nævdal (2016) may be generalized to the case where harvest costs also are strictly convex and in this more general case, the shadow price will also go to infinity as the stock approaches zero. This is done in Proposition 3. Proposition 3 thinks of the stable manifold in a slightly unusual manner. The stable manifold is a continuous mapping from x to μ and it thus makes sense to think of μ as a function of x. We can then use the ratio $\dot{\mu}/\dot{x}$ and steady state conditions to construct a differential equation with boundary conditions (Judd, 1998, Ch.10.7).

Proposition 3. Along the stable manifold $\lim_{x\downarrow 0}\mu(0) = \infty$ holds.

Proof. Let (x_{ss}, μ_{ss}) be the known steady state level of the optimally managed system defined by problem (1). Over the interval [x^*, x_{ss}] one can find the stable manifold by solving the differential equation:

$$\frac{\dot{\mu}}{\dot{x}} = \frac{\mathrm{d}\mu}{\mathrm{d}x} = \frac{(\rho - G'(x))\mu}{G(x) - d^{-1}(\mu)},$$

with the boundary condition $\mu_{ss} = \mu(x_{ss})$. By Proposition 2 there exists an x^* such that $\mu(x^*) = d(0)$. One can therefore find the solution for $\mu(x)$ over the interval [0, x^*] by solving the following differential equation:

$$\frac{\dot{\mu}}{\dot{x}} = \frac{\mathrm{d}\mu}{\mathrm{d}x} = \frac{(\rho - G'(x))\mu}{G(x)}, \quad \mu(x^*) = d(0).$$

Nævdal (2016) showed that this equation has the solution:

$$\mu(x) = \frac{d(0)G(x^*)}{G(x)}\exp\left(-\int_x^{x^*}\frac{\rho}{G(\eta)}d\eta\right) \tag{10}$$

and that $\lim_{x\downarrow 0}\mu(x) = \infty$. The calculations are reproduced in the Appendix.

Propositions 1, 2 and 3 enable us to draw a more complete phase diagram depicted in Fig. 2. It is worthwhile to note that as the stable manifold entails the allowable combinations of x and μ along an optimal path, it may also in fact be interpreted as a function $\mu(x)$ that gives the derivative of the value function, $\mu(x) = V'(x)$. As the value function $V(x)$ clearly must satisfy $V(0) = 0$, the value function can be demonstrated in the phase diagram as the area below the stable manifold as indicated by the shaded area in Fig. 2.

3. Age structured models

We now examine age structured models in order to see if the results from above carry over. In particular, we want to check whether the shadow price goes to infinity as the stock approaches zero, and whether this also implies that no harvesting will occur at low stock levels.

Recent years have seen increased interest in the economics of age structured models and the implications of dropping lumped parameter models (see, e.g., Tahvonen, 2009, and Skonhoft et al., 2012). Typically, the cohort length of a fish stock is measured in one year as reproduction usually occurs on an annual basis and fish species often have a life span of many years. Therefore, to construct a complete age structured model, usually requires several cohorts with year-class specific contribution to recruitment as well as year-class specific natural survival and harvest rates. Indeed, age structured models quickly become

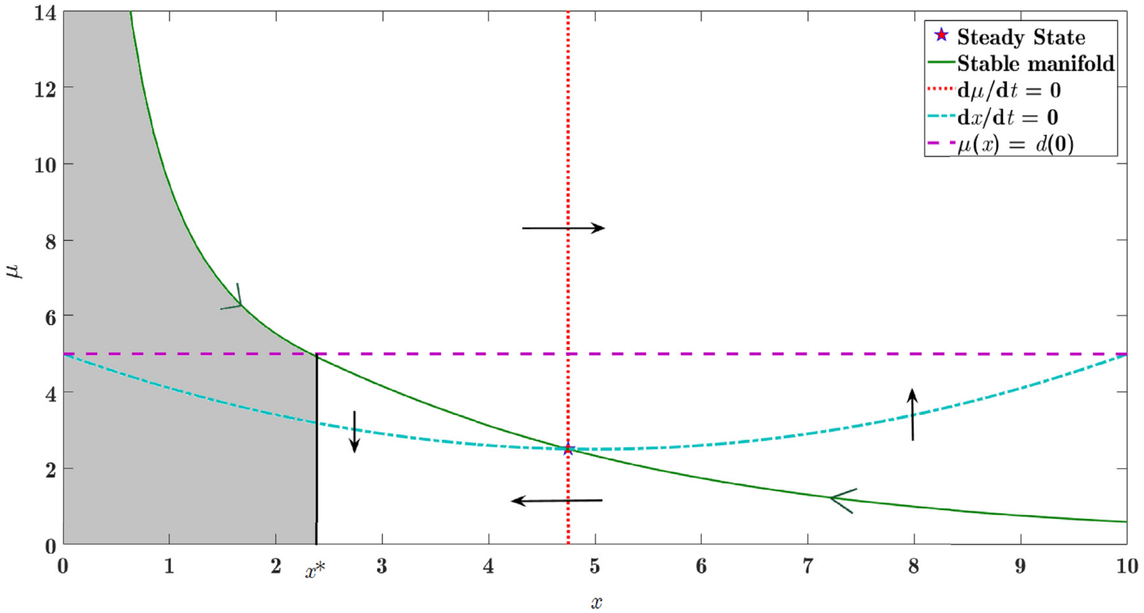

Fig. 2. Computer generated phase diagram for the model in Eq. (1). Note that μ along the stable manifold increases as x goes to zero. As proven in Proposition 3 it does in fact go to infinity. It crosses the line $\mu = d(0)$ at x^*. From Eq. (5) it should be clear that $\mu \geq d(0)$ for $x \leq x^*$ implies that for $x \leq x^*$ we have that $h = 0$. The stable manifold is in fact the derivative of the value function. As $V(0) = 0$, the area under the stable manifold is therefore the value function. The shaded area shows $V(x^*)$, which is the value of the fishery at the stock level x^*. Although Propositions 1-3 hold for general growth and cost functions with the properties stated in the text, the graph is drawn using a logistic growth function $G(x) = rx(1 - x/K)$ with $K = 10$ and $r = 1$. The benefit function $D(h) = ph - C(h) = 5h - ½h^2$, with $p = 5$ as the fixed fish price and $C(h) = ½h^2$ as the cost function. The discount rate has been set to $\rho = 0.05$.

analytically intractable. Here we analyse two simplified cases. One case where the adult period is relatively short compared to the time span of the young fish. This may correspond to e.g. wild Atlantic salmon (*Salmo salar*) where most of the species' life history is in the native river (2–4 years) before it migrates into the ocean and spends 1–2 year there before returning back to spawn in its native river. After spawning it dies (about 90%). The second case we analyse is where the period as a young is short relative to the time (potentially) spent as an adult. This may correspond to e.g., North-East Atlantic cod (*Gadus morhua*) which becomes old enough to spawn at the age of 3 years and may live to become more than 20 years old. In these two particular cases we can use the differences in the time span of cohorts to utilise slow/fast-dynamics in order to simplify the analysis (see, e.g., Crépin, 2007 and Guttormsen et al., 2008).

We explore these two cases within a very simple age structured model with two cohorts, young, x, and adult, y. Both age classes are measured in number of fish, and it is assumed that only adult fish are harvested. The growth equation for young fish is first given by:

$$\dot{x} = F(x, y) - \delta x. \tag{11}$$

Production of young individuals is assumed to depend positively on the stock of adults, but survival of young individuals is assumed to be negatively density dependent. Thus, the recruitment function satisfies $F'_y > 0$ and $F'_x < 0$. Additionally, $F(x, 0) = 0$ must hold as production of young individuals requires adults. Note that this implies $F'_x(x, 0) = 0$ for all x. We shall also assume that all double derivatives of $F(x, y)$ are less than or equal to zero. A fraction δ enters the stock of adults every unit of time. The stock of adults grow according to:

$$\dot{y} = \delta x - \gamma y - h. \tag{12}$$

Here γ is the fixed natural mortality rate and h is again harvesting. We shall assume that there exist a pair $(x, y) = (x_{max}, y_{max})$ defining the steady state when $h = 0$. .

We assume that only adults are harvested. With the applications we have in mind, cod and salmon, assuming harvest of older classes only seems fairly innocuous. It is close to impossible to harvest salmon in any significant numbers before they congregate to spawn. That cod should only be harvested at older age classes seems a fairly robust result, see e.g. Diekert et al. (2010).

3.1. Cod

Here it is assumed that y is the slow variable and x that moves instantaneously from steady state to steady state. Therefore we set $\dot{x} = 0$ and obtain from Eq. (11):

$$\delta x = F(x, y) \Rightarrow x = \varphi(y). \tag{13}$$

From the assumption that $F(x, 0) = 0$ it follows that $\varphi(0) = 0$. Implicit differentiation of (13) yields $\varphi'(y) = -F'_y/(F'_x - \delta) > 0$. Defining $\psi(y) = \varphi(y) - \gamma y$ implies that the expression for $\dot{y}$ may be written as:

$$\dot{y} = \psi(y) - h. \tag{14}$$

Instantaneous benefit from harvesting is given by $D(h)$. It is straight forward to verify that the problem

$$\max_{h \geq 0} \int_0^\infty D(h)e^{-\rho t}dt \quad \text{subject to } \dot{y} = \psi(y) - h, \text{ and } y(0) \text{ given.} \tag{15}$$

has exactly the same structure as (1), so all the propositions from section 2 apply.

3.2. Salmon

Since Atlantic salmon is characterised by a long period as young before experiencing a short period as adult and dying after spawning, we can treat x as a slow variable and y as fast variable. The implication of y being a fast variable is that y moves very quickly from one steady state to another relative to x. Again, as a simplification, we model this by letting the movement of y from one steady state to another be instantaneous implying that $\dot{y} = 0$ or $y = (1/\gamma)(\delta x - h)$from Eq. (12). Inserting into Eq. (11) gives then:

$$\dot{x} = F\left(x, (\delta x - h)\gamma^{-1}\right) - \delta x \tag{16}$$

The management problem may then be written as:

$$\max_{h \geq 0} \int_0^\infty D(h)e^{-\rho t}dt \quad \text{subject to } \dot{y} = F\left(x, (\delta x - h)\gamma^{-1}\right) - \delta x, \ x(0) \text{ given.} \tag{17}$$

We now have an optimization problem with a slightly different structure than in problem (1), so the results from the previous section can not be taken for granted. The Hamiltonian associated with the problem in (17) is given by:

$$H = D(h) + \mu\left(F\left(x, (\delta x - h)\gamma^{-1}\right) - \delta x\right) \tag{18}$$

The Maximum Principle gives the following conditions for optimality:

$$\frac{\partial H}{\partial h} = d(h) - \mu F'_y\left(x, (\delta x - h)\gamma^{-1}\right)\gamma^{-1} \leq 0 \quad (= 0 \text{ if } h > 0) \tag{19}$$

and

$$\dot{\mu} = \rho\mu - \mu\left(F'_x\left(x, (\delta x - h)\gamma^{-1}\right) + \delta F'_y\left(x, (\delta x - h)\gamma^{-1}\right)\gamma^{-1} - \delta\right) \tag{20}$$

Eq. (19) defines optimal harvest as a function of the stock and the shadow price. Let $h(x, \mu)$ be the solution to the equation $\partial H/\partial h = 0$. Then h is defined by:

$$h = \max(0, h(x, \mu)). \tag{21}$$

We can use these conditions to draw a phase diagram. However, in order to draw a correct diagram we must apply some care. We need to draw a line that delineates the state space into regions where $h > 0$ and $h = 0$. From condition (19) we have that for any x, the lowest value of μ such that $h = 0$ is given by

$$\mu = \frac{d(0)}{F'_y(x, \delta x\gamma^{-1})\gamma^{-1}} \tag{22}$$

One way to interpret (22) is that for a given x the stable manifold lies above the μ prescriped by (22), then optimal harvest is zero. If The stable manifold lies below this μ, optimal harvest is positive. The line defined by (22) is illustrated in Fig. 3 as the $h = 0$ line. The isocline where $\dot{x} = 0$ is constructed in the following manner. Inserting $h(x, \mu)$ into Eq. (16) gives:

$$\dot{x} = F\left(x, (\delta x - h(x,\mu))\gamma^{-1}\right) - \delta x = 0 \tag{23}$$

In particular, when $x = 0$, we have that:

$$\dot{x} = F\left(0, -\frac{1}{\gamma}h(0,\mu)\right) = 0. \tag{24}$$

But this is only possible if $h(0, \mu) = 0$ as we have assumed that $F(x, 0) = 0$. It follows that at $x = 0$, the curve $\dot{x} = 0$ intersects the curve $h = 0$.

In order to construct the line for $\dot{\mu} = 0$, we must acknowledge that the shape of this curve depends on whether h is positive or not. If $h(x,\mu) = 0$, then $\dot{\mu} = \rho\mu - \mu(F'_x(x,\delta x\gamma^{-1}) + \delta F'_y(x,\delta x\gamma^{-1})\gamma^{-1} - \delta)$. The equation $\dot{\mu} = 0$ then has two solutions, $\mu = 0$ and $x = \tilde{x}$ defined by $\rho + \delta - (F'_x(\tilde{x},\delta\tilde{x}\gamma^{-1}) + \delta F'_y(\tilde{x},\delta\tilde{x}\gamma^{-1})\gamma^{-1}) = 0$ which is a vertical line. However, when the line $\dot{\mu}$ lies below the $h = 0$ line, the isocline is a curve given by:

$$\dot{\mu} = \rho\mu - \mu\left(F'_x\left(x, (\delta x - h(x,\mu))\gamma^{-1}\right) + \delta F'_y\left(x, (\delta x - h(x,\mu))\gamma^{-1}\right)\gamma^{-1} - \delta\right) = 0 \tag{25}$$

In order to establish a proposition similar to Propositions 1 above we note that in principle we have the same problem. The stable manifold can either intersect the μ-axis where the lines $\dot{x} = 0$ and $h = 0$ intersect in which case $h > 0$ for all $x > 0$, or it can cross the $h = 0$ line at some x^* which implies that $h = 0$ for all $x \in [0, x^*]$. It is a straight forward exercise to confirm that it is the latter that is the case by repeating the Proof in Proposition 1. If the stable manifold starts where the lines $\dot{x} = 0$ and $h = 0$ intersect, then it dips below the line $\dot{x} = 0$ which is a contradiction. Proving the existence of x^* and that $\lim_{x\downarrow 0}\mu(x) = \infty$ is done by verbatim repetition of Propositions 2 and 3.

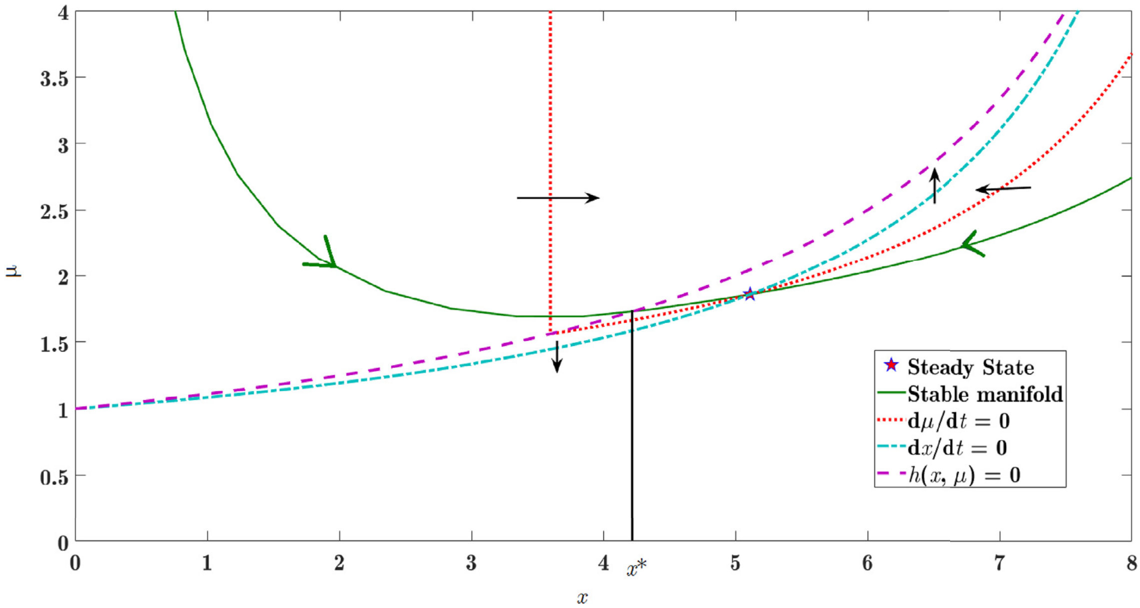

Fig. 3. Phase diagram for cohort fishery with fast slow dynamics. At $x = x^*$, the stable manifold crosses the $h = 0$ boundary. Thus if $x < x^*$ it is optimal to set harvest levels to zero. Also, μ goes to infinity when x goes to zero. The diagram is generated with a version of the growth dynamics where $\dot{\boldsymbol{x}} = ry(1 - x/K) - \delta x$ and $\dot{\boldsymbol{y}} = \delta x - \gamma y - h$. $D(h)$ is again specified as $ph - ½h^2$. Parameter values are given by $r = 1$, $p = 5$, $K = 10$, $\rho = 0.05$, $\delta = 0.15$ and $\gamma = 0.2$.

4. Concluding remarks

In this paper we have examined the basic nonlinear control variable biomass fishery model originating from Clark and Munro (1975), and demonstrated under what circumstances it is optimal to stop harvesting when the stock becomes sufficiently low. The main assumptions in our model of a schooling fishery are: 1, that the intrinsic (maximum) growth rate of the fish stock exceeds that of the discount rent. 2, That the growth of the fish stock is zero when the fish stock is zero and 3. That the marginal net benefit is finite for all harvest levels, and particularly for zero harvest.

The paper provides 3 Propositions and these enable us to draw a more complete phase diagram than what is found in, among others, Clark (2005) and Leonard and Long (1992). The most important of these propositions from a management perspective, is that it always exists a strictly positive stock level below which it is optimal to not harvest. This is perhaps not too surprising. If the value of fish stock grows faster in the ocean than it does in the bank, we would prefer to have the fish staying in the ocean until it has grown to the point where the return in the ocean is equal to returns in the bank. The non-negativity constraint on harvesting implies that we cannot put fish into the lake. However, the fact that optimal harvest levels is always zero for low stock levels also imply that the shadow price of the stock will always go to infinity as the stock goes to zero. This was proven for both the biomass model and for simple cohort models with fast/slow-dynamics.

In a much cited review article, Jim Wilen (2000) points out that the huge literature studying optimal harvesting in fishery models has had negligible impact on actual management in fisheries. He also points out the many reasons for this. However, there have been several attempts to transform optimized dynamic harvest strategies into more practical applicable harvest rules. These harvest control rules (HCR) are typically represented by feedback control rules that links the control variable, the catch or effort, to the state variable, the fish stock. For a review see, e.g., Deroba and Bence (2008). Depending on the formulation of the current benefit function, these HCR can take many forms including the popular proportional harvesting rule; that is, a fixed fraction of the stock should be removed every year. Therefore, this rule allows for harvesting when the stock is close to zero. Our results indicate that harvesting when the stock is close to zero should not be encouraged and that HCR models that do prescribe it as optimal to harvest at close to zero stock levels depend on an assumption of infinite instantaneous marginal benefit of harvesting for this to be correct. Another rule initiated by Engen et al. (1997), is the so-called proportional threshold rule. This HCR indicates that a certain fraction of the fish stock above a certain minimum stock level, the threshold, should be harvested while there should be no harvest at all below the threshold. This harvest rule is accordingly in line with our main finding.

Conflicts of interest

This is to confirm that we in submitting this paper we have no conflicts of interest or financial interests.

Acknowledgements

We are grateful to Colin Clark, Alexander Sigurdsson, Jim Wilen, two anonymous reviewers, Till Requate and participants at the 5th Workshop on Age-structured models in Natural Resource Economics in Kiel for constructive comments. Eric Nævdal is grateful for funding from Norwegian Research Council's program Environment 2015, project 196199.

Appendix. The solution to the differential equation in Proposition 3.

Dividing the differential equation by μ and integrating over $[x, x^*)$ gives:

$$\int_x^{x^*} \frac{1}{\mu(x)} \frac{d\mu}{d\eta} dy = \int_x^{x^*} \frac{\rho}{G(\eta)} d\eta - \int_x^{x^*} \frac{G'(\eta)}{G(\eta)} d\eta$$

$$-\int_{\mu(x^*)}^{\mu(x)} \frac{1}{\mu} d\mu = \int_x^{x^*} \frac{\rho}{G(\eta)} d\eta - \left(\ln\left(G\left(x^*\right)\right) - \ln(G(x))\right)$$

$$\ln\left(\frac{\mu(x)}{\mu\left(x^*\right)}\right) = -\int_x^{x^*} \frac{\rho}{G(\eta)} d\eta + \ln\left(\frac{G\left(x^*\right)}{G(x)}\right)$$

$$\frac{\mu(x)}{\mu\left(x^*\right)} = \frac{G\left(x^*\right)}{G(x)} \exp\left(-\int_x^{x^*} \frac{\rho}{G(\eta)} d\eta\right)$$

Inserting for $\mu\left(x^*\right) = d(0)$ and rearranging gives the expression for $\mu(x)$.

$$\mu(x) = \frac{d(0)G(x^*)}{G(x)} \exp\left(- \int_x^{x^*} \frac{\rho}{G(y)} dy \right)$$

This solution is only valid over the interval $(0, x^*]$. Note that when the integral in this expression converges, $\mu(0)$ is clearly infinite. If the integral does not converge, the expression is of the form "0/0" and must be evaluated with L'Hôpital's rule.

Calculating $\mu(0)$

Applying L'Hôpital's rule yields

$$\lim_{x\to 0} \mu(x) = d(0)G(x^*) \frac{\lim_{x\to 0} \frac{d}{dx}\left(\exp\left(- \int_x^{x^*} \frac{\rho}{G(\eta)} d\eta \right) \right)}{\lim_{x\to 0} G'(x)}$$

$$= d(0)G(x^*) \frac{\lim_{x\to 0} \exp\left(- \int_x^{x^*} \frac{\rho}{G(\eta)} d\eta \right) \frac{\rho}{G(x)}}{\lim_{x\to 0} G'(x)}$$

$$= d(0)G(x^*) \lim_{x\to 0} \frac{\rho}{G'(x)} \times \lim_{x\to 0} \frac{\exp\left(- \int_x^{x^*} \frac{\rho}{G(\eta)} d\eta \right)}{G(x)}$$

$$= \frac{\rho}{G'(0)} \lim_{x\to 0} \frac{d(0)G(x^*)}{G(x)} \exp\left(- \int_x^{x^*} \frac{\rho}{G(\eta)} d\eta \right)$$

The last line implies that:

$$\lim_{x\to 0} \mu(x) = \frac{\rho}{G'(0)} \lim_{x\to 0} \mu(x)$$

This can only be true if $\mu(0) = 0$ or $\mu(0) = \infty$. But because $\dot{\mu} < 0$ in a neighbourhood around $x = 0$ and $G(x) > 0$ it must be true that for x close to zero $\mu'(x) = \dot{\mu}/\dot{x} < 0$, which implies that $\mu(0) = \infty$.

References

Clark, C., 1973. Profit maximization and the extinction of animal species. J. Polit. Econ. 81, 950–961.
Clark, C., Munro, G.R., 1975. The economics of fishing and modern capital theory: a simplified approach. J. Environ. Econ. Manag. 2 (2), 92–106.
Clark, C., 1991. Mathematical Bioeconomics. Wiley Interscience, New York.
Clark, C., 2005. Worldwide Crises in Fisheries. Cambridge UP, Cambridge.
Conrad, J., Clark, C., 1987. Natural Resource Economics. Notes and Problems. Cambridge University Press, Cambridge.
Crépin, A.-S., 2007. Using fast and slow processes to manage resources with thresholds. Environ. Resour. Econ. 36 (2), 91–213.
Cropper, M.L., Lee, D.R., Pannu, S.S., 1979. The optimal extinction of a renewable natural resource. J. Environ. Econ. Manag. 6 (4), 341–349.
Diekert, F.K., Hjermann, D.Ø., Nævdal, E., Stenseth, N.C., 2010. Spare the young fish: optimal harvesting policies for North-East arctic cod. Environ. Resour. Econ. 47 (4), 455–475.
Deroba, J., Bence, J., 2008. A review of harvest policies: understanding relative performance of control rules. Fish. Res. 94, 210–223.
Engen, S., Lande, R., Saether, B.E., 1997. Harvesting strategies for fluctuating populations based on uncertain population estimates. J. Theor. Biol. 186, 201–212.
Guttormsen, A.G., Kristofersson, D., Nævdal, E., 2008. Optimal management of renewable resources with Darwinian selection induced by harvesting. J. Environ. Econ. Manag. 56 (2), 167–179.
Judd, K.L., 1998. Numerical Methods in Economics. MIT Press, Cambridge.
Kamien, M., Schwartz, N., 1991. Dynamic Optimization. North Holland, Amsterdam.
Leung, A., Wang, A., 1976. Analysis of models for commercial fishing: mathematical and economical aspects. Econometrica 44 (2), 295–303.
Leonard, D., Long, N.V., 1992. Optimal Control Theory and Static Optimization in Economics. Cambridge University Press, Cambridge.
Levhari, D., Mirman, L.J., 1980. The great fish war: an example using a dynamic cournot-nash solution. The Bell Journal 11 (1), 322–334.
Lewis, T.R., Schmalensee, R., 1977. Nonconvexity and optimal exhaustion of renewable resources. Int. Econ. Rev. 18 (3), 535–552.
Nævdal, E., 2016. Catastrophes and ex post shadow prices – how the value of the last fish in a lake is infinity and why we shouldn't care (much). J. Econ. Behav. Organ. 132 (Part B), 153–160.
Skonhoft, A., Vestergaard, N., Quaas, M., 2012. Optimal harvest in an age structured model with different fishing selectivity. Environ. Resour. Econ. 51 (4), 525–544.
Sydsæter, K., Hammond, P., Seierstad, A., Strøm, A., 2005. Further Mathematics for Economic Analysis. Prentice Hall, Harlow.
Tahvonen, O., 2009. Economics of harvesting age-structured fish populations. J. Environ. Econ. Manag. 58, 281–299.
Wilen, J., 2000. Renewable resource economists and policy: what differences have we made? J. Environ. Econ. Manag. 39, 306–327.