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Sheep as capital goods and farmers as portfolio managers: a bioeconomic model of Scandinavian sheep farming

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Abstract

A model analyzing the economics of sheep farming is formulated. The basic idea is simple. Sheep are capital and they are held by farmers as long as their capital value exceeds their slaughter, or meat, value. Farmers are therefore portfolio managers aiming to find the optimal combination of different categories of animals. Yields are compared with the yields from other assets. The model is formulated within a Northern Scandinavian economic and biological setting with a crucial distinction between the outdoors grazing season and the indoors season, and with adult sheep and lambs being different categories. In the first step, the management problem is analyzed with only the meat income of the farmers taken into account. In the next step, income from wool production is considered as well. The analysis provides several results that differ from standard harvesting theory.

JEL classifications: C61, D21, Q12, Q29

Keywords: Sheep farming; Stage model; Investment problem

1. Introduction

This article analyzes the economics of sheep farming. The basic idea is simple. Sheep are capital and farmers hold them as long as their capital value exceeds their slaughter, or meat, value. Farmers are, therefore, seen as portfolio managers aiming to find the optimal combination of different categories of animals. Yields are compared with the yields from other assets. This problem has, therefore, similarities with the archetypical renewable natural resource problem (see, e.g., Clark, 1990). However, whereas the standard fishery (or wildlife) problem is formulated in a biomass framework ("a fish is a fish"), the different *age categories* of the sheep asset are central in the following analysis. The study is carried out with a crucial distinction between the outdoor grazing season and the indoor winter season. This is the typical situation found in Northern Scandinavia, and other places in Europe (e.g., mountainous areas in France and Spain) and elsewhere. However, the analysis is essentially related to the economic and biological setting found in Northern Scandinavia and Norway.[1]

There are about 20,000 sheep farms in Norway. These are family farms, and there are around two million animals during the outdoors grazing season. The average farm size, counting about 100 animals in the grazing season, is therefore quite modest and most of the farms are located in mountain areas and other sparsely populated areas; there are also some sheep farms along the coast. The main product is meat, which accounts for about 80% of the average farmer's income. Remaining income comes from wool, as sheep milk production is nonexistent. Housing and indoor feeding is required throughout the winter because of snow and harsh weather conditions. Lambs are born during late winter to early spring, and in early spring the animals usually graze on fenced land. When weather conditions allow, sheep are released into rough grazing areas in the valleys and mountains, which are typically communally owned. The outdoor grazing season ends around late September to the middle of October. Animals are then gathered, wool may be cut and slaughtering takes place. During the summer rough grazing period, flocks may be vulnerable to large predators and to sickness and other injuries (Aunsmo, 1998; Nersten et al., 2003).

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[1] France is an important sheep-producing country, but, in contrast to Northern Scandinavia (see below), milk that is processed to cheese is the main product. Milk production is also the most significant product in Mediterranean countries such as Spain, but meat production from lambs is also important. Wool production is of greatest importance in Sweden and Finland (Aunsmo, 1998).

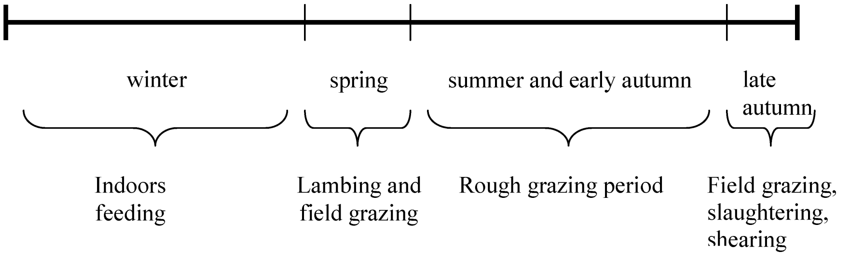

Fig. 1. Seasonal subdivision Northern Scandinavian sheep farming.

Within this system, farmers face several investment decision problems. One problem is to find the optimal size of a farm; that is, the capacity to keep animals indoors during the winter season. Another problem is the so-called replacement problem, i.e., to find the optimal categories, or year classes, of adult females, as fertility (as well as mortality) varies over the life cycle. A third problem is, for a given farm capacity, to find the capacity utilization that gives the optimal number of animals to be fed and kept indoors during the winter season. A corollary of this problem is to find the optimal number of lambs to be slaughtered before the winter season. The main content of this third problem can be studied by considering just two categories, or stages, of the sheep population: lambs and adults. This investment problem is analyzed below, and because only two categories of animals are included, it is possible to solve the problem within a simple optimal control framework. The goal of the analysis is, first, to show the basic economic principles steering the investment problem and, next, to relate the outcome of this harvesting model to the standard harvesting theory model pioneered in the work by Clark (1990). As will be demonstrated, there are similarities, but also some important differences.

Caswell (2001) and Getz and Haigh (1989) provide in-depth analysis of stage models of fish and wildlife. However, economic analysis plays only a minor role in these works. Among the few stage models with economic content is the two-stage seal model of Conrad and Bjørndal (1991). On the other hand, there is extensive literature on the economics of livestock management (see, e.g., Jarvis, 1974; Kennedy, 1986), but most of this literature has little relevance for a farming system with a distinct seasonal subdivision. The problem of the typical cow-calf operator in the western United States has some similarities with the Scandinavian sheep farming system, but the problem here is typically to determine the length of the grazing season, in addition to determine the stocking level (see, e.g., Huffaker and Wilen, 1991). In contrast to this, the length of the grazing season is fixed in our model. There are several papers that analyze the replacement problem and consider the different categories of the adult sheep (see, e.g., Avramita et al., 1981). Typically, these models are large linear programming-type models. Fisher (2001) is an example of a detailed linear programming model that analyzes the economics of what are called a spring lambing system, a winter lambing system, and an accelerated lambing system in Canada. In the following analysis, the spring lambing scheme is taken for granted because of the climatic conditions. It is also assumed that the outdoors grazing conditions represent no constraint on the size of the flock and on the growth of the animals; this problem is studied in a companion paper. On the other hand, as already indicated, winter farm capacity is assumed fixed. Relaxing this assumption is also analyzed in the companion paper.

2. The biological model

The biological model is formulated in discrete time with a seasonal subdivision between the outdoor grazing period (spring, summer, and fall) and indoor feeding period (winter). The sheep population is structured (e.g., Caswell, 2001) as adult females, and young females and males, henceforth called lambs. Lambs are born in late winter to early spring, just before the grazing season starts. Lambs not slaughtered enter the adult population after the slaughtering period (i.e., September to October). All male lambs are assumed slaughtered since only very few (or none when artificial insemination is practiced) are kept for breeding. Therefore, only female adults are considered. Fertility is assumed fixed. Natural mortality differs between adults and lambs and is fixed and density independent as well.[2] All natural mortality is assumed to occur during the grazing season. Demographic data on sheep are available in Mysterud et al. (2002).

When stochastic variations in biology and environment are ignored, the number of adult females in year $(t + 1)$ just after slaughtering is made up of the previous year's adults surviving natural mortality and not slaughtered and the female lambs surviving natural mortality and not slaughtered (see Fig. 1). This may be written as $X_{1,t+1} = X_{1,t}(1 - m_1)(1 - h_{1,t}) + X_{0,t}(1 - m_0)(1 - h_{0,t})$, where $X_{0,t}$ is the number of female lambs, m_1 and m_0 are the mortality fractions of adult females and lambs, respectively, and $h_{1,t}$ and $h_{0,t}$ are the fractions slaughtered.[3] With the fecundity rate f (lambs per adult female) and the same number of male and female lambs being recruited, $X_{0,t} = 0.5 f X_{1,t}$ yields the number of female lambs. Therefore, the adult female

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[2] In reality, fecundity may be improved (and mortality may be reduced) at a cost. However, such possible trade-offs are not included in this article.

[3] New animals from outside may be added, but this possibility is ignored in the present exposition.

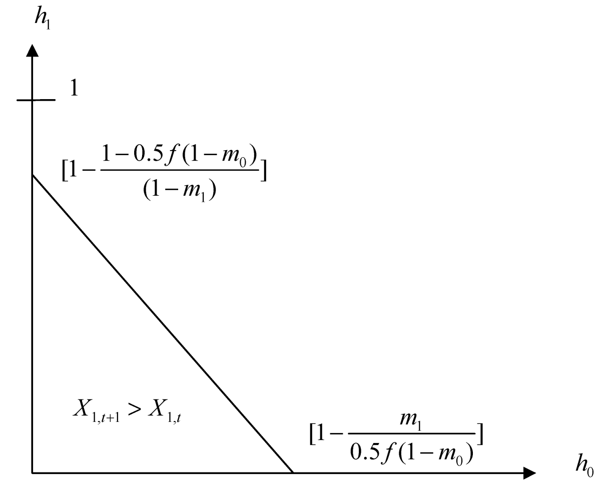

Fig. 2. Steady-state harvesting relationship Eq. (2) (no predation). h_0 female lamb slaughtering fraction, h_1 adult slaughtering fraction.

population growth is:

$$X_{1,t+1} = X_{1,t}(1-m_1)(1-h_{1,t}) + 0.5fX_{1,t}(1-m_0)(1-h_{0,t}). \tag{1}$$

In a linear growth model such as this, in contrast to the standard fishery (and wildlife) density dependent model (e.g., Clark, 1990), it is well known that, with no removal of animals, i.e., $h_{1,t} = h_{0,t} = 0$, the population will either grow without bound or die out (see Caswell, 2001).[4] However, with slaughtering, an infinite combination of harvesting fractions can sustain a stable population. Such steady-state harvesting rates are found when $X_{1,t+1} = X_{1,t} > 0$, and may be written as:

$$h_1 = 1 - \frac{1-0.5f(1-m_0)(1-h_0)}{(1-m_1)}. \tag{2}$$

Equation (2), see also Fig. 2, describes a downward sloping line in the (h_0, h_1) plane, and a constant population can hence be sustained with either a "low" h_0 and a "high" h_1, or the opposite. Harvesting combinations outside this line lead to a shrinking population, whereas combinations inside the line lead to growth. Condition (2) intersects with the h_1-axis at $[1-[1-0.5f(1-m_0)]/(1-m_1)]$, which may be above or below one. Therefore, the highest adult-harvesting rate compatible with the steady state is min $\{1, [1-[1-0.5f(1-m_0)]/(1-m_1)]\}$. For all realistic parameter values, it will be below one (see numerical section), and only this situation is considered (but see footnote 7). Equation (2) intersects with the h_0-axis at $[1-m_1/0.5f(1-m_0)] < 1$ and is the highest lamb-harvesting rate compatible with the steady state. Not surprisingly, these maximum values increase with higher fertility and lower mortality.

3. Revenue and costs

Up to this point, we have ignored any income from wool production. The sale of meat is the only revenue component. Because slaughtering takes place after natural mortality, the number of adult animals removed is $H_{1,t} = X_{1,t}(1-m_1)h_{1,t}$. The number of slaughtered female lambs is $HF_{0,t} = 0.5fX_{1,t}(1-m_0)h_{0,t}$, and the entire male lamb subpopulation is removed (see above), $HM_{0,t} = 0.5fX_{1,t}(1-m_0)$. Let p_1 and p_0 be the net (of slaughtering cost) adult and lamb slaughtering prices, in euro per animal. We assume these to be constant over time and independent of the number of animals supplied at the farm level. Meat income for year t is then:

$$Q_t = p_1X_{1,t}(1-m_1)h_{1,t} + p_0 0.5fX_{1,t}(1-m_0)(h_{0,t}+1). \tag{3}$$

The cost structure differs sharply between the outdoor grazing season and the indoor feeding season. Generally, the indoor costs are substantial higher. The length of the indoor season is determined by climate conditions and is therefore exogeneously given. Throughout this analysis, it is assumed that farm capacity is fixed.[5] Therefore, the costs of buildings, machinery, and so forth are constant, and given by γ. The indoor season variable costs include labor cost (typically as an opportunity cost), electricity, and veterinarian costs, in addition to fodder and vary with the given length of the indoors season. Variable costs increase with the size of the winter population and, as the capacity constraint is approached, these costs may rise steeply. This is approximated by a convex function, and the winter total cost function is specified as $CW_t = \gamma + (\beta/2)X_{1,t}^2$ with the parameter $\beta > 0$.

As indicated, during the grazing period sheep essentially graze on communally owned lands (commons). In Norway, such land is usually available without cost. There may be some transportation and maintenance costs, which altogether are assumed to be linearly related to the size of the grazing flock as follows $CS_t = \alpha\,(X_{1,t} + fX_{1,t})$, with $\alpha > 0$, when measured before natural mortality. Therefore, ignoring discounting within the year, the yearly cost is:

$$C_t = \gamma + \alpha(1+f)X_{1,t} + (\beta/2)X_{1,t}^2. \tag{4}$$

The difference between this cost structure and that of the standard resource economic model (e.g., Clark, 1990) is that while operating cost is typically decreasing in stock size in the standard model (due to a lower unit harvesting cost), it is increasing in the present model. However, we often find a similar cost structure in management models of terrestrial wildlife resources where the species are a nuisance and damage, for instance, on crop production (e.g., Skonhoft, 1999; Swanson, 1994).

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[4] It is easily recognized that the population will die out if $m_1 > 0.5f(1-m_0)$. Therefore, with removal of animals, the demographic parameters must be scaled such that $f > m_1/0.5(1-m_0)$.

[5] The problem of also allowing for physical capital accumulation and changing farm capacity is progressively more difficult to analyze because one has to account for irreversibility (see the pioneering work of Clark et al., 1979 in a fishery context). As mentioned, this problem is taken up in an accompanying paper.

4. The optimal program

We assume the farmer aims to maximize the present value of profit over an infinite time horizon, $\sum_{t=0}^{\infty} \rho^t [Q_t - C_t]$ under the biological constraint (1). $\rho = 1/(1+\delta)$ is the discount factor with $\delta \geq 0$ as the (yearly) discount rate.[6] Inserted from Eqs. (3) and (4), the current-value Hamiltonian of this problem is (cf. Conrad and Clark, 1987)

$$\begin{aligned}\Psi = {} & p_1 X_{1,t}(1-m_1)h_{1,t} + p_0 0.5 f X_{1,t}(1-m_0)(h_{0,t}+1) \\ & - [\gamma + \alpha(1+f)X_{1,t} + (\beta/2)X_{1,t}^2] \\ & + \rho\lambda_{t+1}[X_{1,t}(1-m_1)(1-h_{1,t}) \\ & + 0.5 f X_{1,t}(1-m_0)(1-h_{0,t}) - X_{1,t}],\end{aligned}$$

where $\lambda_t > 0$ is the resource shadow price. The first-order conditions with $X_{i,t} > 0$ and the adult-harvesting fraction below one (but see below) yield:

$$\partial\Psi/\partial h_{1,t} = p_1 - \rho\lambda_{t+1} \leq 0; \quad h_1 \geq 0, \tag{5}$$

$$\partial\Psi/\partial h_{0,t} = p_0 - \rho\lambda_{t+1} \leq 0; \quad h_0 \geq 0, \tag{6}$$

and

$$\begin{aligned}-\partial\Psi/\partial X_t = {} & -p_1(1-m_1)h_{1,t} - p_0 0.5 f(1-m_0)(h_{0,t}+1) \\ & + \alpha(1+f) + \beta X_{1,t} - \rho\lambda_{t+1}[(1-m_1)(1-h_{1,t}) \\ & + 0.5 f(1-m_0)(1-h_{0,t}) - 1] = \rho\lambda_{t+1} - \lambda_t.\end{aligned} \tag{7}$$

The interpretation of control condition (5) is that adult slaughtering should take place up to the point where the marginal meat income is equal to, or below, the cost of reduced growth in stock numbers, evaluated at the shadow price. Following the Kuhn-Tucker theorem, it holds with equality when the removal of this subpopulation is optimal at the steady state. The lamb control condition (6) has the same interpretation; that is, lamb slaughter should occur up to the point where the marginal meat income is equal to, or below, the cost of reduced growth in stock numbers, evaluated at the shadow price. This condition also holds with equality when the removal of this subpopulation is optimal at the steady state. Equation (7) is the portfolio condition, which essentially states that the number of adult females should be maintained so that natural growth equals the shadow price of growth, adjusted for the discount factor. With the assumption that the steady state is reachable from the given initial value $X_{1,0}$, the dynamics will typically follow the Most Rapid Approach Path (*MRAP*) as the Hamiltonian is linear in the controls, and hence will involve a "bang-bang" control. Accordingly, if the stock is above that of the steady state, it should be slaughtered down the first year. In the opposite situation with too few animals, it is optimal to postpone slaughtering until the steady state is reached.

At the steady state, the shadow price is fixed through the control conditions. However, these conditions cannot be generally satisfied simultaneously as equations, and except when prices are equal, only one of the categories should be harvested. Therefore, we have to distinguish between three cases: i) if $p_0 > p_1$, lamb-only slaughtering is optimal; ii) if $p_0 < p_1$, adult-only slaughtering is optimal; and iii) if $p_0 = p_1$, both categories should be slaughtered. The argument for case i) is simply that if steady state lamb-only slaughtering is beneficial, condition (6) should hold as equality and condition (5) as an inequality. It is easily recognized that this demands $p_0 > p_1$. The argument for case (ii) follows in a similar manner. This is stated as:

Result 1. For $p_0 \neq p_1$, *only one-stage slaughtering is optimal at the steady state.*

A corollary to Result 1 and the biological equilibrium condition (2) is:

Result 2. When $p_0 \neq p_1$, slaughtering should take place at the highest level compatible with the steady state.

The main content of this investment problem hence boils down to a simple principle, and one-stage slaughtering only results because the harvest benefit is linear in both harvest activities. This is the same reason for the "bang-bang" nature of the solution (see above). On the other hand, the reason for slaughtering at the highest level compatible with steady state follows from the lack of any density dependent effects in the biological model. At the same time, this means that the harvest rate depends only on biological conditions. Therefore, when the lamb price is above that of the adult price, which fits reality and is considered as the main case (cf. numerical section), the case (i) steady-state harvesting rates are:

$$h_1^* = 0 \tag{8}$$

and

$$h_0^* = 1 - \frac{m_1}{0.5 f(1-m_0)}. \tag{9}$$

The accompanying shadow price is $\lambda^* = p_0/\rho = p_0(1+\delta)$. At the steady-state, the resource shadow price hence exceeds the market value of the harvest (when $\delta > 0$). If the portfolio condition (7) is evaluated at the steady state and the shadow price is replaced, we find that the harvest rate disappears. Therefore, after small rearrangements, the golden rule condition may be written as:

$$(1+\delta) = f(1-m_0) + (1-m_1) - \frac{\alpha(1+f) + \beta X_1}{p_0}. \tag{10}$$

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[6] The realism of present-value maximizing as a management goal may be questioned for various reasons. However, it can be shown that maximizing current profit under the condition of equilibrium harvesting, $X_{1,t+1} = X_{1,t}$, yields the same solution as the steady state of present-value maximization for a zero discount rate, $\delta = 0$ (see the main text below). Therefore, the steady-state flock size for a zero discount rate is similar to the flock size when current-value profit is maximized for a stable population.

This condition states that the internal rate of return, comprising the natural growth rate adjusted for the cost–price ratio, should equal the external rate $(1 + \delta)$, and Eq. (10) alone determines the unique solution of the steady-steady number of adult animals X_1^*. Note that economic parameters, in addition to biological parameters, influence X_1^*. This is in line with standard harvest theory.

In the opposite case (ii) of $p_0 < p_1$, the steady-state harvesting rates are $h_1^* = [1 - [1 - 0.5f(1 - m_0)]/(1 - m_1)] < 1$ and $h_0^* = 0$. Following the same steps as above, it is straightforward to show that the golden rule condition in this case is $(1 + \delta) = [(p_0 + p_1)/p_1\,]0.5f(1 - m_0) + (1 - m_1) - [\alpha\,(1 + f) + \beta\,X_1^*\,]/p_1$.

Hence, the harvesting rate is still not included in the golden rule condition, but both prices influence the internal rate of return in this adult-only harvesting case.[7] The lamb harvesting price influences the optimal flock size when there is no lamb slaughtering because of the biologically "indirect" nature of this stage; lambs this year are slaughtered as adults next year.

Case (iii), with $p_0 = p_1 = p$, means that both categories should be harvested at the steady state, $h_0^* > 0$ and $h_1^* > 0$. After some rearrangements, it can be shown that the portfolio condition is just as the above Eq. (10), except that p replaces p_0. As the control conditions (5) and (6) now yield the same information, there is one degree of freedom in the system of equations and unknowns. Accordingly, either the lamb- or the adult-harvesting rate must be given exogenously. Hence, all harvesting rates along the equilibrium-harvesting schedule (2) are equally beneficial for the farmer.

5. Changes in the economic and biological environment

The above analysis demonstrates that the meat prices alone determine the harvesting decision, whereas biological conditions alone determine the harvest rate. On the other hand, biological as well as economic factors determine the optimal adult flock size to be kept during the winter, and hence the optimal outdoor grazing lamb population. When considering the main case (i) where $p_0 > p_1$ and rewriting the golden rule condition (10), the adult steady-state flock size becomes:

$$X_1^* = \frac{p_0[f(1-m_0)+(1-m_1)-(1+\delta)]-\alpha(1+f)}{\beta}. \qquad (10')$$

Not surprisingly, permanent higher mortality rates means a smaller flock size. The fertility rate effect is, on the other hand, positive when the lamb marginal harvesting income dominates the marginal outdoors season cost; that is, when $p_0(1 - m_0) > \alpha$. This must hold to secure a positive steady state profit. Higher fertility also means a higher harvesting rate (Eq. 9), and it can easily be shown that the equilibrium profit $(Q^* - C^*) = p_0\ 0.5fX_1^*\,(1 - m_0)(h_0^* + 1) - [\gamma + \alpha\,(1 + f)X_1^* + (\beta/2)X_1^{*2}]$ increases as well. This fits intuitive reasoning as higher fertility is to be considered as a cost-free gift of Mother Nature (but see note 2).

More costly farming, either during the indoor feeding season, β, or the grazing season, α, results in fewer animals. The discount rate has the standard negative stock effect as well, whereas condition (10′) clearly indicates a positive price effect, $\partial X_1^*/\partial p_0 > 0$. This is stated as:

Result 3. A higher slaughter price results in a larger steady state flock size.

This result contrasts with what is found in the standard harvesting model (Clark, 1990). The reason is, however, straightforward as there is no stock-dependent harvesting, or slaughtering, cost (see also above). Therefore, a higher p_0 simply means that it becomes relatively less expensive to keep animals as both α/p_0 and β/p_0 decrease (see also Skonhoft, 1999 and Swanson, 1994). With more adult animals, there will also be more grazing lambs, $X_0^* = 0.5fX_1^*$. The optimal number of removed animals (cf. the above Section 3), consisting of lambs only (female and male), is $HF_0^* + HM_0^* = 0.5fX_1^*\,(1 - m_0)(h_0^* + 1) = X_1^*\,[f(1 - m_0) - m_1\,]$. Therefore, we find that slaughtering increases with a higher harvesting price as well because X_1^* increases.

6. Extending the basic model

So far, income from wool production has been ignored. However, wool income can be significant for some farmers and, as indicated, it contributes about 20% of the total sheep farm income in Northern Scandinavia (e.g., Aunsmo 1998). Following today's practice, the farmer may choose whether to shear the sheep once or twice a year. If the fleece is shorn once, this will be in spring and for adults only. Therefore, the lambs that survive natural mortality and slaughtering are not shorn before they are one year old. In the other case of shearing two times a year, there is an additional shearing just before slaughtering. The last scheme is considered here, as this is the most common practice (Aunsmo, 1998). The yearly wool income is then written as $W_t = q[\sigma_s X_{1,t} + \sigma_a(1 - m_1)X_{1,t} + \tau f X_{1,t}(1 - m_0)]$, where q is the net (net of shearing costs) wool price (euro per tonne wool), σ_s and σ_a are the (average) per unit adult spring and autumn outputs (tonne per animal), respectively, and τ is the per lamb output. This may be simplified to:

$$W_t = q\theta X_{1,t}, \qquad (11)$$

where $\theta = \sigma_s + \sigma_a(1 - m_1) + \tau f(1 - m_0)$ is the demographic and seasonally adjusted per unit output coefficient. Accordingly, adding wool implies joint production, meat and wool, of the fixed coefficient type.

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[7] If the biological conditions are such that $h_1^* = 1$ (which, as mentioned, is unrealistic due to the actual biological parameter values), the lamb-harvesting rate compatible with the steady state follows from Eq. (2) as $h_0^* = [1 - 1/0.5f(1 - m_0)]$. In this special version of case ii), the complementary slack conditions change and the above control condition (5) reads $p_1 - \rho\lambda > 0$ at the steady state. Moreover, as lambs are slaughtered as well, (6) holds as an equation at the steady state, $p_0 - \rho\lambda = 0$. These conditions are consistent with $p_0 < p_1$.

With W_t included as part of the farm income, a stock effect is added to the harvesting decision. This provides a mechanism similar to the "wealth effect" in models of optimal growth (see the classic Kurz, 1968 article). When maximizing present-value profit including wool income, $\sum_{t=0}^{\infty} \rho^t [Q_t - C_t + W_t]$, under the biological constraint (1), it follows directly that the control conditions (5) and (6) stay unchanged because the control variables, the harvest fractions, are not included in the new term W_t. This is stated as:

Result 4. Including a stock value leaves the harvesting decision unchanged, and the harvesting fractions stay unchanged as well.

Because adding a stock value changes the relative valuation between keeping an asset and selling it, this result also contrasts with standard harvesting theory and intuitive reasoning. It holds irrespective of the profitability of wool production, q, whether the per-lamb wool output coefficient is "low" or "high" and whether there is a price difference between adult and lamb wool (which is not considered here). On the other hand, the portfolio condition (7) changes as it is extended with the marginal wool income term ("wealth effect"). In the lamb-only harvesting case (i), it can be verified that the steady-state adult flock size now becomes:

$$X_1^* = \frac{p_0[f(1-m_0) + (1-m_1) - (1+\delta)] + q\theta - \alpha(1+f)}{\beta}. \tag{12}$$

The effect $\partial X_1^*/\partial q > 0$ is easily recognized. As a corollary of this effect and $\partial h_0^*/\partial q = 0$, the number of animals removed increases, $\partial(HF_0^* + HM_0^*)/\partial q > 0$. Hence, in contrast to the expected result that the farmer should sell less of an asset that becomes more valuable, the farmer sells more. This is stated as:

Result 5. Including a stock value leads to more animals being slaughtered.

The model may also be extended to include predation. During the grazing period, sheep in Northern Scandinavia are vulnerable to predation from four big predators: bears (*Ursus arctos*), wolverines (*Gulo gulo*), wolves (*Canis lupus*), and lynxes (*Lynx lynx*). Although the total loss is modest (yearly, less than 1% of the sheep population is reported lost), farmers in a few areas may be seriously affected. With r_0 and r_1 as the fixed lamb and adult predation rates, respectively, assumed to be purely additive to natural mortality, the population growth Eq. (1) changes to[8] $X_{1,t+1} = X_{1,t}(1-m_1)(1-r_1)(1-h_{1,t}) + 0.5fX_{1,t}(1-m_0)(1-r_0)(1-h_{0,t})$. Accordingly, the new harvesting equilibrium schedule shifts downward relative to the previous condition (2), $h_1 = 1 - [1 - 0.5f(1-m_0)(1-r_0)(1-h_0)]/[(1-m_1)(1-r_1)]$.

With predation, the meat income of the farmer also changes and reads $\tilde{Q}_t = p_1 X_{1,t}(1-m_1)(1-r_1)h_{1,t} + p_0 0.5fX_{1,t}(1-m_0)(1-r_0)(h_{0,t}+1)$.

When the present-value profit is maximized under the new biological growth equation, we find the same control conditions as above, i.e., (5) and (6) hold irrespective of the relative predation pressure on adults and lambs. However, the harvesting fraction falls, and the lamb-only harvesting case (i) now yields:

$$h_0^* = 1 - \frac{1-(1-m_1)(1-r_1)}{0.5f(1-m_0)(1-r_0)}. \tag{13}$$

Therefore, predation on lambs, but also on adults, reduces the harvesting rate. This is stated as:

Result 6. Predation leaves the slaughtering decision unchanged. The optimal harvesting fraction is reduced.

When the wool income term again is ignored, the steady-state female adult flock size becomes $X_1^* = (1/\beta)\{p_0[f(1-m_0)(1-r_0) + (1-m_1)(1-r_1) - (1+\delta)] - \alpha(1+f)\}$. Not surprisingly, the flock size is reduced compared with the no-predation situation, $\partial X_1^*/\partial r_i < 0$ $(i = 0, 1)$. Consequently, the number of animals slaughtered and meat income are reduced.

Above, it is tacitly assumed that the farmer receives no economic compensation for the predation loss. However, compensation is normally paid by the State (Environment Department, 2003). If natural mortality is assumed to take place before predation (the opposite will not change the results qualitatively), the number of adults and lambs lost through predation is $R_{1,t} = X_{1,t}(1-m_1)r_1$ and $R_{0,t} = fX_{1,t}(1-m_0)r_0$, respectively. With $k_i > 0 (i = 0, 1)$ as the per unit compensation value, the yearly compensation is $K_t = k_1 X_{1,t}(1-m_1)r_1 + k_0 fX_{1,t}(1-m_0)r_0$, which may be simplified to:

$$K_t = \phi X_{1,t}, \tag{14}$$

where $\phi = k_1(1-m_1)r_1 + k_0 f(1-m_0)r_0$ is the demographic adjusted per adult compensation value.

Current profit is now $(\tilde{Q}_t - C_t + K_t)$ (when wool income is ignored), and present-value profit maximization, given the new biological growth equation including predation, yields the same control conditions as above. Therefore, when considering the lamb-only harvesting case (i) with $p_0 > p_1$, we find the same harvesting fraction as under predation without compensation (Eq. 13), but lower than that without predation. However, the

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[8] The assumption of fixed predation rates reflects the situation of a constant predation pressure through time, e.g., a fixed number of wolves through time. As the predation loss increases linearly with sheep density, assuming constant predation rates (see main text below), the ecological interaction is consistent with the famous Lotka–Volterra predator–prey model (see, e.g., Clark, 1990).

Table 1
Steady-state different value categories included. Lamb only harvesting case ($p_0 > p_1$). X_1^* number adult animals, h_0^* female lamb slaughtering fraction, ($HF_0^* + HM_0^*$) total lamb slaughtering and π^* profit (in euro).

	X_1^*	h_0^*	$HF_0^* + HM_0^*$	π^*
No wool production. No predation. Baseline price p_0 (Eqs. 9 and 10′)	119	0.93	160	8,218
No wool production. No predation. 25% price increase p_0 (Eqs. 9 and 10′)	155	0.93	208	13,876
Wool production. No predation. Baseline price p_0 (Eqs. 9 and 12)	138	0.93	181	10,503
Predation with full compensation. No wool production. Baseline price p_0 (Eqs. 13 and 15)	118	0.88	147	8,149

Note: Fixed costs neglected.

optimal steady state population size changes to:

$$X_1^* = \frac{p_0[f(1-m_0)(1-r_0) + (1-m_1)(1-r_1) - (1+\delta)] + \phi - \alpha(1+f)}{\beta}. \tag{15}$$

Predation with compensation leads to more animals than without predation (Eq. 10) if $\{p_0\ [f(1-m_0)(1-r_0) + (1-m_1)(1-r_1)] + \phi\} > p_0\ [f(1-m_0) + (1-m_1)]$. This inequality may also be written as $(\phi/p_0) > [f(1-m_0)r_0 + (1-m_1)r_1\ ]$. In principle, the farmer should be exactly compensated (Environment Department, 2003).[9] With $k_i = p_i (i = 0, 1)$, the adjusted per adult compensation value becomes $\phi = p_1(1-m_1)r_1 + p_0\ f(1-m_0)r_0$. Inserted into the above inequality, we hence find $[p_1(1-m_1)r_1 + p_0\ f(1-m_0)r_0\ ]/p_0 > [f(1-m_0)r_0 + (1-m_1)r_1]$ which reduces to $p_1 > p_0$. This inequality is, however, violated under the lamb-only harvesting scheme of $p_0 > p_1$. Therefore, even if the farmer is fully compensated, it is optimal to keep a smaller flock size and slaughter fewer animals than to farm without predation.

More interesting, however, is that the profit will be reduced as well. The reason for this is twofold. First, and most important, predation of both categories of animals means that adult animals are "harvested" as well. This is not optimal under the given price structure. Second, predation imposes an additional constraint on the slaughtering decision of the profit-maximizing farmer. With predation, the farmer must "harvest" twice a year and the first harvest, i.e., the predation, takes place with predation rates fixed by Mother Nature. So, even when the price per animal of this "harvest" is the same as under the regular slaughtering, the profit will decrease compared with the situation of no such constraint. The numerical simulations confirm this reasoning. This is stated as:

Result 7. Predation with full compensation yields a smaller flock and fewer animals slaughtered than without predation. Profit is reduced.

7. Numerical illustration

To shed some further light on the above analysis, the model is illustrated numerically. The Appendix gives the data used in the simulations and Table 1 reports the results of the lamb-only harvesting case (i).

Without wool production and predation, the optimal lamb-only harvesting fraction is 0.93 (Result 1 and Result 2) and the optimal adult steady-state flock size is 119 animals. If the lamb price shifts up 25%, the adult flock size increases to 155 animals (Result 3) and the number of lambs slaughtered increases from 160 to 208. With wool production and the baseline lamb meat price, but still no predation, both the adult flock size and number of animals slaughtered increase (Result 5) and the harvesting fraction stays unchanged (Result 4). Predation with full compensation, but without wool production, gives a small reduction in the number of adult animals compared with the no wool and no predation scenario. The harvesting fraction is more affected, as it falls to 0.88 (Result 6). Profit declines, albeit slightly, irrespective of the full compensation scheme, from 8,218 to 8,149 (Result 7).

8. Concluding remarks

This article has analyzed the economics of sheep farming in a two-stage model of lambs and adult females. The analysis presented at the farm level, in a Northern Scandinavian context. A crucial distinction is made between the outdoor grazing season and the winter indoor feeding season. Meat production is the basic product. Farm capacity is assumed given, and the problem is to find the optimal number of animals to be kept indoors during the winter and the number of animals to be slaughtered before the winter season when it is assumed that the farmer aims to maximize present-value profit. The article demonstrates the economic principles steering this investment problem, and the findings are related to standard bioeconomic harvest theory.

In this two-stage model of lambs and adult females, it is demonstrated that the harvesting decision is determined by economic factors alone and, for the given price structure, lamb-only harvesting is the best strategy. On the other hand, the optimal lamb harvest fraction depends only on biological factors. In line with the standard theory, the optimal flock size is determined jointly by biological and economic factors. The basic

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[9] However, in reality, the amount of compensation may differ because of various reasons, such as cheating and overestimating the loss.

reason for this sharp distinction between the effects of economic and biological forces is the lack of any density-dependent factors regulating sheep population growth. Including wool income in addition to meat value, and including predation during the grazing season, leaves this structure more or less unchanged. Several results that contrast standard harvest theory are provided.

Because only two categories of animals are considered, the model aims not to predict the farming practice in detail. However, the main results of the model replicate today's situation where farmers basically slaughter and sell lamb meat (Aunsmo, 1998). Within our model, the only factor explaining this practice is that lamb meat is more valuable than adult meat. Therefore, a crucial part of the farmer's portfolio management problem boils down to a very simple decision rule. On the other hand, how much capital to hold is progressively more difficult to answer as economic as well as biological factors determine the optimal population size. The baseline numerical example indicates a lamb-harvesting fraction of 0.93. This is above the actual harvesting fraction, which averages about 0.75 in Norway (Aunsmo, 1998). The low adult mortality assumption (0.05) explains most of this difference. In reality, however, adult mortality is higher and it differs sharply between different age classes of animals. A more complete model, with many categories of animals and where mortality, as well as fertility, vary with age (the replacement model, cf. Section 1), may give a solution where slaughtering of some adult categories are optimal. Thus, if adult mortality is increased somewhat following a more complete model, our results would be more in accordance with reality. Finally, when including predation together with full compensation for the loss of animals, the model shows that the farmer's profit will fall compared to the no predation situation. However, the numerical illustration indicates that such loss may be quite modest.

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Appendix: Data and parameter values

The Appendix reports the base-line data used in the simulations. The biological parameter values are based on Aunsmo (1998) and Mysterud et al. (2002). Aunsmo and Nersten et al. (2003) provide economic data, but some of the key parameters are calibrated to ensure realistic stock size. The variable marginal winter cost is crucial here. The fixed cost component is omitted because of lack of data. All prices are in 2003 value.

Table A1
Baseline values prices and costs, ecological parameters and other parameters

Parameter	Parameter description	Value
m_0	-Natural mortality fraction lambs	0.09
m_1	-Natural mortality fraction adult	0.05
f	-Fertility rate	1.53 (lamb per adult)
r_0	-Predation fraction lambs	0.05
r_1	-Predation fraction adult	0.03
p_0	-Lamb slaughter price	120.0 (euro per animal)
p_1	-Adult slaughter price	100.0 (euro per animal)
α	-Fixed marginal cost summer grazing	10.5 (euro per animal)
β	-Variable marginal cost winter	1.1 (euro per animal)
q	-Wool price	4,300 (euro per tonne)
θ	-Adjusted wool output coefficient	0.005 (tonne per animal)
δ	-Discount rent	0.03

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